C1 Binomial Expansion

Specimen

**7.** *(a)* $(2x+3)^4 = (2x)^4 + 4(2x)^3(3) + 6(2x)^2(3)^2 + 4(2x)(3)^3 + 3^4$ — M1 (5 terms, 2 correct); A2 (unsimplified, −1 for each error)

$= 16x^4 + 96x^3 + 216x^2 + 216x + 81$ — A1 (simplified)

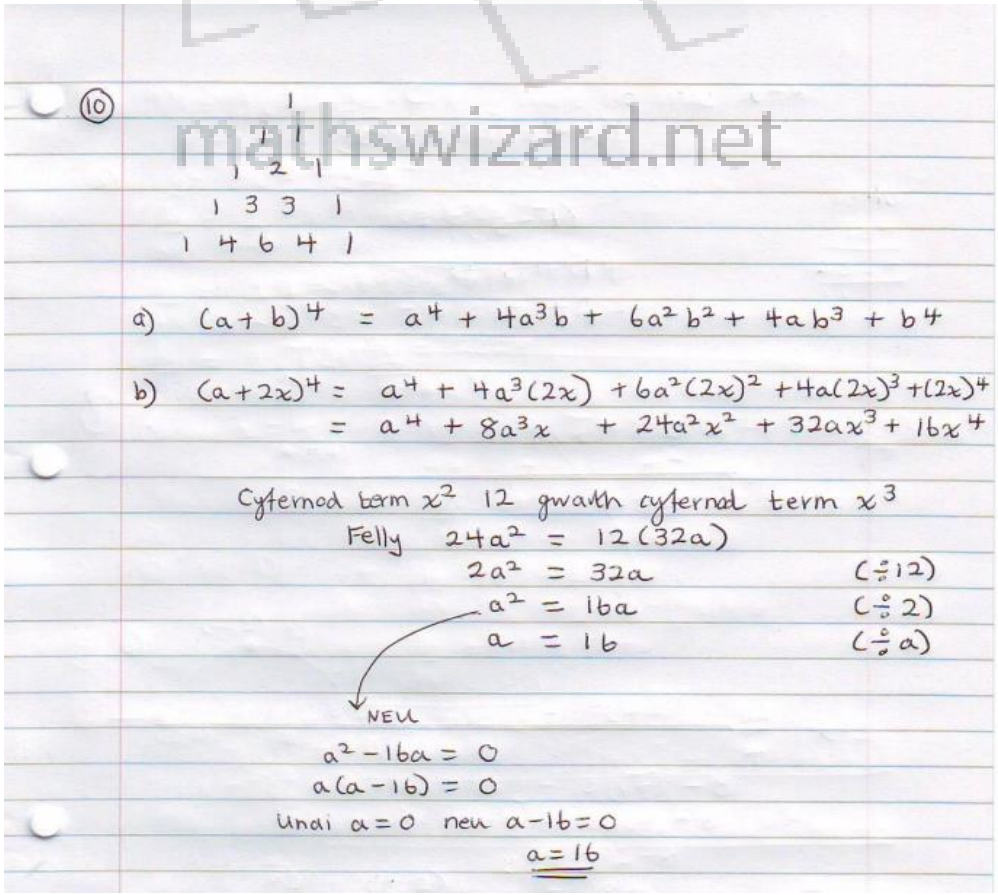
*(b)* ${}^nC_2(3)^2 = 54$ — B1

so that $\frac{n(n-1)3^2}{1.2} = 54$ — B1

$\therefore n^2 - n - 12 = 0$ — M1 (standard form of quadratic and attempt to solve)

$\therefore n = 4$ — A1

2005 Winter



(10)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

a) $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

b) $(a+2x)^4 = a^4 + 4a^3(2x) + 6a^2(2x)^2 + 4a(2x)^3 + (2x)^4$
$= a^4 + 8a^3x + 24a^2x^2 + 32ax^3 + 16x^4$

Cyfernod term $x^2$ 12 gwaith cyfernod term $x^3$
Felly $24a^2 = 12(32a)$
$2a^2 = 32a$ (÷12)
$a^2 = 16a$ (÷2)
$a = 16$ (÷a)

NEU
$a^2 - 16a = 0$
$a(a-16) = 0$
Unai $a = 0$ neu $a - 16 = 0$
$a = 16$

2005 Summer

**4.** $(1+2x)^6 = 1+6(2x)+\frac{6.5}{1.2}(2x)^2+\frac{6.5.4}{1.2.3}(2x)^3+.....$ M1 (substitution of $2x$, $n$=6 in $(1+x)^n$)

$=1+12x+60x^2+160x^3+.....$ A1 $(1+12x)$
A1 $(60x^2)$
A1 $(160x^3)$

[4]

2006 Winter

**7.** *(a)* $(3x+2)^3 = (3x)^3+3(3x)^2(2)+3(3x)(2)^2+2^3$

$=27x^3+54x^2+36x+8$ B3 (–1 for each error, any method)

*(b)* $\frac{n(n-1)}{2}(2)^2 = 2(2n)$ M1 ($^nC_2\,2^p = 2k^nC_1$, $k$=2, $\frac{1}{2}$, $p$ = 1,2)

A2 ($\frac{n(n-1)}{2}2^2$=2(2$n$))

$n=3$ A1 (C.A.O.)

7



2006 Summer

**6.** *(a)* $(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$ B1

An attempt to substitute $3x$ for $a$ and $\pm\frac{1}{3}$ for $b$ in r. h. s. of above expansion M1

Required expression $= 81x^4-36x^2+6-\frac{4}{9x^2}+\frac{1}{81x^4}$

(3 terms correct) A1
(all terms correct) A2
(f.t. one slip in coefficients of $(a+b)^4$ )

*(b)* Either: $\frac{n(n-1)}{2}\times 2^k = 40(k=1,2)$

Or: $^nC_2\times 2^2 = 40$ M1
$n=5$ A1

2007 Winter

**4.** (*a*) $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ (–1 for each error)
(–1 for any subsequent 'simplification') B2

(*b*) $(2+x)^4 = 2^4 + 4\times(2)^3\times x + 6\times(2)^2\times x^2 + 4\times(2)\times x^3 + x^4$
(f.t. for at least 4 terms, not all coefficients equal to 1)
$(2+x)^4 = 16 + 32x + 24x^2 + 8x^3 + x^4$ (–1 for each error) B2
An attempt to collect terms and form quadratic equation M1
$16 + 32x + 24x^2 + 8x^3 + x^4 = 14 + 33x + 25x^2 + 8x^3 + x^4$
$\Rightarrow x^2 + x - 2 = 0 \Rightarrow (x+2)(x-1) = 0 \Rightarrow x = -2, 1$ (c.a.o.) A1

2007 Summer

**5.** (*a*) $(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$
(–1 for each error)
(–1 for any subsequent 'simplification') B2
Substituting $x$ for $a$ and $\frac{1}{2x}$ for $b$ in the $10a^3b^2$ term M1
Identifying $\frac{5}{2}$ (o.e.) as the coefficient of $x$ A1

(*b*) Coefficient of $x^2 = \frac{n(n-1)}{2}$ B1
An attempt to solve $\frac{n(n-1)}{2} = 36$ M1
$n = 9$ (c.a.o.) A1



2008 Winter

**4.** (*a*) $(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$ (-1 for each error)
(-1 for any subsequent 'simplification') B2

(*b*) (i) $\left(1+\frac{x}{2}\right)^5 \approx 1^5 + 5\left(\frac{x}{2}\right)1^4 + \frac{5(5-1)}{2}\left(\frac{x}{2}\right)^2 1^3 + \frac{5(5-1)(5-2)}{2\times 3}\left(\frac{x}{2}\right)^3 1^2$
Two terms correct B1
Other two terms correct B1
$\left(1+\frac{x}{2}\right)^5 \approx 1 + \frac{5(x)}{2} + \frac{10(x)^2}{4} + \frac{10(x)^3}{8}$ B1

(ii) An attempt to substitute $x = 0{\cdot}1$ in candidate's expression for $\left(1+\frac{x}{2}\right)^5$ M1
$1{\cdot}05^5 \approx 1{\cdot}276(25)$ (c.a.o.) A1

2008 Summer

**6.** $(5 + 2x)^3 = 125 + 150x + 60x^2 + 8x^3$

Two terms correct B1
Three terms correct B1
All 4 terms correct B1

2009 Winter

**6.** (*a*) $(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$ (–1 for each error)
(–1 for any subsequent 'simplification') B2

(*b*) An expression containing $k \times (1/4)^2 \times (2x)^3$, where $k$ is an integer $\neq 1$ and is either the candidate's coefficient for the $a^2b^3$ term in (*a*) or is derived from first principles M1
Coefficient of $x^3 = 5$ (c.a.o.) A1



2009 Summer

**7.** (*a*) $\left(x+\frac{2}{x}\right)^4 = x^4 + 4x^3\left(\frac{2}{x}\right) + 6x^2\left(\frac{2}{x}\right)^2 + 4x\left(\frac{2}{x}\right)^3 + \left(\frac{2}{x}\right)^4$

(three terms correct) B1
(all terms correct) B2

$\left(x+\frac{2}{x}\right)^4 = x^4 + 8x^2 + 24 + \frac{32}{x^2} + \frac{16}{x^4}$ (three terms correct) B1

(all terms correct) B2

(– 1 for incorrect further 'simplification')

(*b*) A correct equation in $n$, including ${}^nC_2 = 55$ M1
$n = 11, -10$ (c.a.o.) A1
$n = 11$ (f.t. $n = 10$ from $n = -11, 10$) A1

2010 Winter

**7.** Coefficient of $x = {}^5C_1 \times a^4 \times 3(x)$ B1

Coefficient of $x^2 = {}^5C_2 \times a^3 \times 3^2(x^2)$ B1

$10 \times a^3 \times m = k \times 5 \times a^4 \times 3$ (o.e.) ($m = 9$ or $3$, $k = 8$ or $^1/_8$) M1

$a = \frac{3}{4}$ (c.a.o.) A1

2010 Summer

**4.** (*a*) $(1 + x)^6 = 1 + 6x + 15x^2 + 20x^3 + \ldots$

All terms correct B2

Three terms correct B1

(*b*) An attempt to substitute $x = -0{\cdot}01$ (or $x = -0{\cdot}1$) in the expansion of part (*a*) (f.t. candidate's coefficients from part (*a*)) M1

$(0{\cdot}99)^6 \approx 1 - 6 \times 0{\cdot}01 + 15 \times 0{\cdot}0001 - 20 \times 0{\cdot}000001$

(At least three terms correct, f.t. candidate's coefficients from part (*a*)) A1

$(0{\cdot}99)^6 = 0{\cdot}94148 = 0{\cdot}9415$ (correct to four decimal places) (c.a.o.) A1



2011 Winter

**5.** $(1 + \sqrt{3})^5 = (1)^5 + 5(1)^4(\sqrt{3}) + 10(1)^3(\sqrt{3})^2 + 10(1)^2(\sqrt{3})^3 + 5(1)(\sqrt{3})^4 + (\sqrt{3})^5$

(five or six terms correct) B2

(four terms correct) B1

$(1 + \sqrt{3})^5 = 1 + 5\sqrt{3} + 30 + 30\sqrt{3} + 45 + 9\sqrt{3}$

(six terms correct) B2

(four or five terms correct) B1

$(1 + \sqrt{3})^5 = 76 + 44\sqrt{3}$ (f.t. one error) B1

2011 Summer

**7.** (*a*) $(3+2x)^4 = 3^4 + 4\times3^3\times(2x) + 6\times3^2\times(2x)^2 + 4\times3\times(2x)^3 + (2x)^4$

(all terms correct) B2

(three or four terms correct) B1

$(3+2x)^4 = 81 + 216x + 216x^2 + 96x^3 + 16x^4$

(all terms correct) B2

(three or four terms correct) B1

(–1 for incorrect further 'simplification')

(*b*) Coefficient of $x = {}^nC_1 \times \frac{1}{4}(x)$ B1

Coefficient of $x^2 = {}^nC_2 \times \frac{1}{4^2}(x^2)$ B1

$\frac{n(n-1)}{2} \times \frac{1}{4^m} = k \times n \times \frac{1}{4}$ (o.e.) ($m = 1$ or 2, $k = 5$ or $^1/_5$) M1

$n = 41$ (c.a.o.) A1



2012 Winter

**4.** (*a*) $\left[x + \frac{3}{x}\right]^4 = x^4 + 4x^3\left[\frac{3}{x}\right] + 6x^2\left[\frac{3}{x}\right]^2 + 4x\left[\frac{3}{x}\right]^3 + \left[\frac{3}{x}\right]^4$

(all terms correct B2)

(3 or 4 terms correct B1)

$\left[x + \frac{3}{x}\right]^4 = x^4 + 12x^2 + 54 + \frac{108}{x^2} + \frac{81}{x^4}$

(all terms correct B2)

(3 or 4 terms correct B1)

(– 1 for further incorrect simplification)

(*b*) ${}^nC_2 \times 2^k = 760$ ($k = 1, 2$) M1

Either $2n^2 - 2n - 760 = 0$ or $n^2 - n - 380 = 0$ or $n(n-1) = 380$ A1

$n = 20$ (c.a.o.) A1

2012 Summer

**4.** $(1-2x)^6 = 1 - 12x + 60x^2 - 160x^3 + \ldots$ B1 B1 B1 B1

(– 1 for further incorrect simplification)

2013 Winter

**7.** Coefficient of $x = {}^6C_1 \times a^5 \times 4(x)$ B1

Coefficient of $x^2 = {}^6C_2 \times a^4 \times 4^2(x^2)$ B1

$15 \times a^4 \times m = k \times 6 \times a^5 \times 4$ ($m$ = **16** or 4 or 8, $k$ = **2** or $^1/_2$) M1

$a = 5$ (c.a.o.) A1



2013 Summer

**5.** (*a*) $(1+2x)^7 = 1 + 14x + 84x^2 \ldots$ B1 B1 B1

(*b*) $(1-4x)(1+2x)^7 = 1 - 4x + 14x - 56x^2 + 84x^2$

Constant term and terms in $x$ B1

Terms in $x^2$ B1

(f.t. candidate's expression in (*a*))

$(1-4x)(1+2x)^7 = 1 + 10x + 28x^2$ (c.a.o.) B1

2014 Winter

**5.** (*a*) $(1+\sqrt{6})^5 = (1)^5 + 5(1)^4(\sqrt{6}) + 10(1)^3(\sqrt{6})^2 + 10(1)^2(\sqrt{6})^3$
$+ 5(1)(\sqrt{6})^4 + (\sqrt{6})^5$ (five or six terms correct) B2
**(If B2 not awarded**, award B1 for four correct terms)
$(1+\sqrt{6})^5 = 1 + 5\sqrt{6} + 60 + 60\sqrt{6} + 180 + 36\sqrt{6}$
(six terms correct) B2
**(If B2 not awarded**, award B1 for four or five correct terms)
$(1+\sqrt{6})^5 = 241 + 101\sqrt{6}$ (f.t. one error) B1

(*b*) ${}^nC_2 \times 3^k = 495$ ($k = 1$, **2**) M1
Either $9n^2 - 9n - 990 = 0$ or $n^2 - n - 110 = 0$ or $n(n-1) = 110$ A1
$n = 11$ (c.a.o.) A1

2014 Summer

**4.** (*a*) $(1+x)^6 = 1 + 6x + 15x^2 + 20x^3 + \ldots$
All terms correct B2
**If B2 not awarded,** award B1 for three correct terms

(*b*) An attempt to substitute $x = 0{\cdot}1$ in the expansion of part (*a*)
(f.t. candidate's coefficients from part (*a*)) M1
$1{\cdot}1^6 \approx 1 + 6 \times 0{\cdot}1 + 15 \times 0{\cdot}01 + 20 \times 0{\cdot}001$
(At least three terms correct, f.t. candidate's coefficients from part (*a*))
A1
$1{\cdot}1^6 \approx 1{\cdot}77$ (c.a.o.) A1



2015

**6.** (*a*) $\left[1 - \frac{x}{2}\right]^8 = 1 - 4x + 7x^2 - 7x^3 + \ldots$ B1 B1 B1 B1
(– 1 for further incorrect simplification)

(*b*) First term $= 2^n$ B1
$2^n = 32 \Rightarrow n = 5$ B1
Second term $= n \times 2^{n-1} \times ax$ B1
$a = -3$ (f.t. candidate's value for $n$) B1

2016

**4.** $(\sqrt{3}-1)^5 = (\sqrt{3})^5 + 5(\sqrt{3})^4(-1) + 10(\sqrt{3})^3(-1)^2 + 10(\sqrt{3})^2(-1)^3$
$+ 5(\sqrt{3})(-1)^4 + (-1)^5$ (five or six terms correct) B2

**(If B2 not awarded**, award B1 for three or four correct terms)

$(\sqrt{3}-1)^5 = 9\sqrt{3} - 45 + 30\sqrt{3} - 30 + 5\sqrt{3} - 1$ (six terms correct) B2

**(If B2 not awarded**, award B1 for three, four or five correct terms)

$(\sqrt{3}-1)^5 = -76 + 44\sqrt{3}$ (f.t. one error) B1

2017

**5.** (*a*) $\left[x + \frac{2}{x}\right]^4 = x^4 + 4x^3\left[\frac{2}{x}\right] + 6x^2\left[\frac{2}{x}\right]^2 + 4x\left[\frac{2}{x}\right]^3 + \left[\frac{2}{x}\right]^4$

(4 or 5 terms correct) B2

**(If B2 not awarded**, award B1 for 3 correct terms)

$\left[x + \frac{2}{x}\right]^4 = x^4 + 8x^2 + 24 + \frac{32}{x^2} + \frac{16}{x^4}$ (5 terms correct) B2

**(If B2 not awarded**, award B1 for 3 or 4 correct terms)

(– 1 for further incorrect simplification)

(*b*) Coefficient of $x = {}^6C_1 \times a^5 \times 2(x)$ B1

Coefficient of $x^2 = {}^6C_2 \times a^4 \times 2^2(x^2)$ B1

$15 \times a^4 \times m = 6 \times a^5 \times 2$ ($m$ = **4** or 2) M1

$a = 5$ (c.a.o.) A1