C1 Completing the Square Answers

Specimen

**5.** *(a)* $2(x^2 - 6x) + 25$

$= 2[(x-3)^2 - 9] + 25$

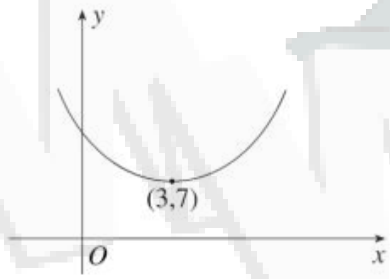
$= 2(x-3)^2 + 7$ B1(*a*), B1(*b*), B1(*c*)

(*a*, *b*, *c* need not be displayed).

*(b)* Least value of $2x^2 - 12x + 25 = 7$ B1

Corresponding value of $x = 3$ B1

*(c)*




M1 (shape)
A1 (location of minimum)

2005 Winter



(5) $x^2 - 14x + 55$

$= (x-7)^2 - 49 + 55$

$= (x-7)^2 + 6$

Felly $a = 7$, $b = 6$

Mae'n rhaid bod $x^2 - 14x + 55$ yn bositif ar gyfer pob gwerth o $x$ oherwydd gwerth lleiaf y ffwythiant yw 6 (rhaid cael $(x-7)^2 \geq 0$ ar gyfer pob gwerth o $x$).

2005 Summer

| | | | |
|---|---|---|---|
| **8.** | (a) | $x^2 - 16x + 16 = (x - 3)^2 + 7$ | B1 ($(x - 3)^2$)<br>B1 (7) (b) |
| | | Least value = 7 | B1 (F.T. candidate's $b$, least val must be mentioned) |
| | | No marks for answer derived by calculus. | |
| | (b) | $x^2 + 2x + 1 \leq 4x + 9$ | M1 (correct method of rearranging) |
| | | $x^2 - 2x - 8 \leq 0$ | |
| | | $(x - 4)(x + 2) \leq 0$ | A1 (fixed points 4, –2 identified, C.A.O.) |
| | | $-2 \leq x \leq 4$ | M1 (any method)<br>A1 (F.T. fixed points) |
| | | Allow B1 for $x \geq x \geq 4$ | M1 (any method)<br>A1 (F.T. fixed points) |
| | | Allow B1 for $x \geq -2$, or $x \leq 4$<br>MO AO for $x \leq 4$ and $x \leq -2$ | |

[7]



2006 Winter

| | | | |
|---|---|---|---|
| **9.** | *(a)* | $23 + 6x - x^2 = 32 - (x - 3)^2$ | B1 ($-(x - 3)^2$<br>B1 (32) |
| | | Greatest value = 32<br>when $x = 3$ | B1 (F.T. candidate's $b$ and $a$)<br>B1 |
| | *(b)* | $\frac{1}{30 + 6x - x^2} = \frac{1}{7 + 23 + 6x - x^2}$ | A1 |
| | | Least value = $\frac{1}{39}$ | A1 (F.T. $b$, $a$) |

## 2006 Summer

| | | | | |
|---|---|---|---|---|
| **4.** | (*a*) | An expression for $b^2 - 4ac$, with $b = \pm 4$, and at least one of $a$ or $c$ correct | | M1 |
| | | $b^2 - 4ac = 4^2 - 4k(k - 3)$ | | A1 |
| | | $b^2 - 4ac = 4(k - 4)(k + 1)$ | | A1 |
| | | Putting $b^2 - 4ac = 0$ | | m1 |
| | | $k = -1, 4$ | (f.t. one slip) | A1 |
| | (*b*) | $a = 4$ | | B1 |
| | | $b = -14$ | | B1 |
| | | Least value $= -14$ | (f.t. candidate's $b$) | B1 |

## 2007 Winter

| | | | | |
|---|---|---|---|---|
| **8.** | (*a*) | $a = 2$ | | B1 |
| | | $b = 5$ | | B1 |
| | | Maximum value $= \frac{1}{5}$ | (f.t. candidate's $a$ and $b$) | B2 |
| | (*b*) | $x + 2 = x^2 - 5x + 11$ | | M1 |
| | | An attempt to collect terms, form and solve quadratic equation | | m1 |
| | | $(x - 3)^2 = 0 \Rightarrow$ curve and line touching | | A1 |
| | | $x = 3, y = 5$ | | A1 |
| | | **Special case** | | |
| | | Differentiating and equating to get $1 = 2x - 5$ | | M1 |
| | | $x = 3$ | | A1 |
| | | $y = 5$ (from one equation) | | A1 |
| | | Verification that $x = 3, y = 5$ satisfies other equation | | A1 |



## 2007 Summer

| | | | |
|---|---|---|---|
| **7.** | (*a*) | $a = 2$ | B1 |
| | | $b = 1$ | B1 |
| | | $c = 3$ | B1 |
| | (*b*) | $\frac{1}{c+4}$ on its own or maximum value $= \frac{1}{c+4}$, with correct explanation or no explanation | B2 |
| | | $\frac{1}{c+4}$ on its own or maximum value $= \frac{1}{c+4}$, with incorrect explanation | B1 |
| | | minimum value $= \frac{1}{c+4}$ with no explanation | B1 |
| | | minimum value $= \frac{1}{c+4}$ with incorrect explanation | B0 |
| | | **Special case** | |
| | | Candidates who give maximum value $= \frac{1}{3}$ are awarded B1 | |

2008 Winter

| | | | |
|---|---|---|---|
| **7.** | $p = 0{\cdot}9$ | | B1 |
| | A convincing argument to show that the value 4 is correct | | B1 |
| | $x^2 + 1{\cdot}8x - 3{\cdot}19 = 0 \Rightarrow (x + 0{\cdot}9)^2 = 4$ | | M1 |
| | $x = 1{\cdot}1$ | | A1 |
| | $x = -2{\cdot}9$ | | A1 |

2008 Summer

| | | | |
|---|---|---|---|
| **5.** | (*a*) | $a = 3$ | B1 |
| | | $b = -13$ | B1 |
| | (*b*) | $2b$ on its own or least (minimum) value = $2b$, with correct explanation or no explanation | B1 |
| | | $x = -a$ | B1 |
| | | **Note:** Candidates who use calculus are awarded B0, B0 | |

2009 Winter

| | | |
|---|---|---|
| **4.** | $a = 3$ | B1 |
| | $b = -2$ | B1 |
| | $c = 5$ | B1 |
| | A positive quadratic graph | M1 |
| | Minimum point $(-b, c)$ | A1 |



2009 Summer

| | | | | | |
|---|---|---|---|---|---|
| **4.** | (*a*) | (i) | $a = -2{\cdot}5$ | (or equivalent) | B1 |
| | | | $b = 1{\cdot}75$ | (or equivalent) | B1 |
| | | (ii) | Greatest value $= -b$ | (or equivalent) | B1 |
| | (*b*) | $x^2 - x - 7 = 2x + 3$ | | | M1 |
| | | An attempt to collect terms, form and solve quadratic equation | | | m1 |
| | | $x^2 - 3x - 10 = 0 \Rightarrow (x - 5)(x + 2) = 0 \Rightarrow x = 5, x = -2$ | | (both values, c.a.o.) | A1 |
| | | When $x = 5$, $y = 13$, when $x = -2$, $y = -1$ | | (both values f.t. one slip) | A1 |
| | | The line $y = 2x + 3$ intersects the curve $y = x^2 - x - 7$ at the points $(-2, -1)$ and $(5, 13)$ | | (f.t. candidate's points) | E1 |

2010 Winter

| | | | |
|---|---|---|---|
| **4.** | (*a*) | $a = 4$ | B1 |
| | | $b = -1$ | B1 |
| | | $c = 3$ | B1 |
| | (*b*) | $\frac{1}{c}$ on its own or greatest value = $\frac{1}{c}$, with correct explanation or no explanation | B2 |
| | | **If B2 not awarded** | |
| | | $\frac{1}{c}$ on its own or greatest value = $\frac{1}{c}$, with incorrect explanation | B1 |
| | | least value = $\frac{1}{c}$ with no explanation | B1 |
| | | least value = $\frac{1}{c}$ with incorrect explanation | B0 |

2

2010 Summer

| | | | |
|---|---|---|---|
| **5.** | (*a*) | $a = 2$ | B1 |
| | | $b = 3$ | B1 |
| | | $c = -25$ | B1 |
| | (*b*) | $6x^2 + 36x - 17 = 3\,[a(x + b)^2 + c] + k$ ($k \neq 0$, candidate's $a$, $b$, $c$) | M1 |
| | | Least value = $3c + 4$ (candidate's $c$) | A1 |



2011 Winter

| | | |
|---|---|---|
| **6.** | Either $p = -0{\cdot}7$ or a sight of $(x - 0{\cdot}7)^2$ | B1 |
| | A convincing argument to show that the value 9 is correct | B1 |
| | $x^2 - 1{\cdot}4x - 8{\cdot}51 = 0 \Rightarrow (x - 0{\cdot}7)^2 = 9$ | M1 |
| | $x = 3{\cdot}7$ | A1 |
| | $x = -2{\cdot}3$ | A1 |

2011 Summer

| | | | |
|---|---|---|---|
| **4.** | $a = -3$ | | B1 |
| | $b = 2$ | | B1 |
| | A negative quadratic graph | | M1 |
| | Maximum point (3, 2) | (f.t. candidate's values for $a$, $b$) | A1 |

2012 Winter

**5.** (*a*) $a = 3$ B1

$b = -1$ B1

$c = 2$ B1

(*b*) An attempt to substitute 1 for $x$ in an appropriate quadratic expression (f.t. candidate's value for $b$) M1

Maximum value = $^1/_8$ (c.a.o.) A1

2012 Summer

**5.** (*a*) $a = 3$ B1

$b = -2$ B1

$c = 17$ B1

(*b*) Stationary value = 17 (f.t. candidate's value for $c$) B1

This is a minimum B1

2013 Winter

**4.** (*a*) (i) $a = 4$ B1

$b = -11$ B1

(ii) least value $-33$ (f.t. candidate's value for $b$) B1

corresponding $x$-value = $-4$ (f.t. candidate's value for $a$) B1

(*b*) $x^2 - x - 9 = 2x - 5$ M1

<u>An attempt to collect terms, form and use a correct method to solve their quadratic equation</u> m1

$x^2 - 3x - 4 = 0 \Rightarrow (x - 4)(x + 1) = 0 \Rightarrow x = 4, x = -1$ (both values, c.a.o.) A1

When $x = 4, y = 3$, when $x = -1, y = -7$ (both values, f.t. one slip) A1

The line [$y = 2x - 5$] intersects the curve [$y = x^2 - x - 9$] at two points ($-1, -7$) and (4, 3) (f.t. candidate's x and $y$-values) E1

2013 Summer

**4.** (*a*) $2(x-4)^2-40$ B1 B1 B1

(*b*) least value = – 20 (f.t. candidate's value for *c*) B1
$x$-coordinate = 4 (f.t. candidate's value for *b*) B1

2014 Winter

**4.** Either $p = 0{\cdot}8$ or a sight of $(x + 0{\cdot}8)^2$ B1
A convincing argument to show that the value 25 is correct B1
$x^2 + 1{\cdot}6x - 24{\cdot}36 = 0 \Rightarrow (x + 0{\cdot}8)^2 = 25$ (f.t. candidate's value for *p*) M1
$x = 4{\cdot}2$ (f.t. candidate's value for *p*) A1
$x = -5{\cdot}8$ (f.t. candidate's value for *p*) A1

2014 Summer

**5.** (*a*) $a = 4$ B1
$b = -1$ B1
$c = 7$ B1

(*b*) An attempt to substitute 1 for $x$ in an appropriate quadratic expression (f.t. candidate's value for *b*) M1
Greatest value of $\dfrac{1}{4x^2-8x+29} = \dfrac{1}{25}$ (c.a.o.) A1



2015

**4.** (*a*) $4(x-3)^2-225$ B1 B1 B1

(*b*) $4(x-3)^2 = 225$ (f.t. candidate's values for *a, b, c*) M1
$(x-3) = (\pm)\dfrac{15}{2}$ (f.t. candidate's values for *a, b, c*) m1
$x = \dfrac{21}{2}, -\dfrac{9}{2}$ (both values) A1

2016

**5.** (*a*) $a = 2$, $b = -12$ B1 B1

(*b*) $x^2 + 4x - 8 = 2x + 7$ M1

An attempt to collect terms, form and solve the quadratic equation in $x$ either by correct use of the quadratic formula or by writing the equation in the form $(x + n)(x + m) = 0$, where $n \times m$ = candidate's constant m1

$x^2 + 2x - 15 = 0 \Rightarrow (x - 3)(x + 5) = 0 \Rightarrow x = 3, x = -5$

(both values, c.a.o.) A1

When $x = 3$, $y = 13$, when $x = -5$, $y = -3$

(both values, f.t. one slip) A1

2017

**4.** (*a*) $a = -2$ B1

$b = 5$ B1

$c = 85$ B1

(*b*) Stationary value = 85 (f.t. candidate's value for $c$) B1

This is a maximum B1