C1 Completing the Square Questions

Specimen

5. *(a)* Express $2x^2 - 12x + 25$ in the form $a(x - b)^2 + c$, where a, b, c are constants to be determined. [3]

(b) Find the least value of $2x^2 - 12x + 25$ and the corresponding value of x. [2]

(c) Sketch the curve $y = 2x^2 - 12x + 25$. [2]

2005 Winter

5. Express the quadratic expression $x^2 - 14x + 55$ in the form $(x - a)^2 + b$, where the values of the constants a and b are to be determined. Hence show that $x^2 - 14x + 55$ is positive for all values of x. [5]

2005 Summer

8. *(a)* Express the quadratic expression $x^2 - 6x + 16$ in the form $(x - a)^2 + b$, where the values of the constants a and b are to be determined. **Deduce** the least value of $x^2 - 6x + 16$. [3]

(b) Solve the inequality

$$(x + 1)^2 \leqslant 4x + 9.$$ [4]

2006 Winter

9. *(a)* Express $23 + 6x - x^2$ in the form $b - (x - a)^2$, where the constants a and b are to be determined. Hence find the greatest value of $23 + 6x - x^2$ and the corresponding value of x. [4]

(b) Use the results found in *(a)* to deduce the least value of $\frac{1}{30 + 6x - x^2}$. [2]

2006 Summer

4. *(a)* Given that the equation

$$kx^2 - 4x + k - 3 = 0$$

has equal roots, find the values of k. [5]

(b) Express $x^2 + 8x + 2$ in the form $(x + a)^2 + b$. Hence write down the least value of $x^2 + 8x + 2$. [3]

2007 Winter

8. *(a)* Express $x^2 + 4x + 9$ in the form $(x + a)^2 + b$, where the values of a and b are to be determined.
Deduce the maximum value of

$$\frac{1}{x^2 + 4x + 9}.$$

[4]

(b) Show that the line $y = x + 2$ touches the curve $y = x^2 - 5x + 11$, and find the coordinates of the point of contact. [4]

2007 Summer

7. *(a)* Express $2x^2 + 4x + 5$ in the form $a(x + b)^2 + c$, where a, b and c are to be determined. [3]

(b) Use the result derived in *(a)* to find the greatest value of $\frac{1}{2x^2 + 4x + 9}$. [2]

2008 Winter

7. Show that $x^2 + 1{\cdot}8x - 3{\cdot}19$ may be expressed in the form $(x + p)^2 - 4$, where p is a constant whose value is to be found.
Hence solve the quadratic equation $x^2 + 1{\cdot}8x - 3{\cdot}19 = 0$. [5]

2008 Summer

5. *(a)* Express $x^2 + 6x - 4$ in the form $(x + a)^2 + b$ where the values of a, b are to be determined. [2]

(b) **Use your results to part *(a)*** to find the least value of $2x^2 + 12x - 8$ and the corresponding value of x. [2]

2009 Winter

4. Express $3x^2 - 12x + 17$ in the form $a(x + b)^2 + c$, where the values of the constants a, b and c are to be found.
Hence, sketch the graph of $y = 3x^2 - 12x + 17$, indicating the coordinates of its stationary point. [5]

2009 Summer

4. *(a)* (i) Express $x^2 - 5x + 8$ in the form $(x + a)^2 + b$, where the values of the constants a and b are to be found.

(ii) Deduce the greatest value of $-x^2 + 5x - 8$. [3]

(b) Solve the simultaneous equations $y = x^2 - x - 7$ and $y = 2x + 3$ algebraically. Write down a geometrical interpretation of your results. [5]

2010 Winter

4. *(a)* Express $4x^2 - 8x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants whose values are to be found. [3]

(b) Use your answer to part *(a)* to find the greatest value of

$$\frac{1}{4x^2 - 8x + 7}.$$ [2]

2010 Summer

5. *(a)* Express $2x^2 + 12x - 7$ in the form $a(x + b)^2 + c$, where the values of the constants a, b and c are to be found. [3]

(b) **Use your answer to part** *(a)* to find the least value of $6x^2 + 36x - 17$. [2]

2011 Winter

6. Show that $x^2 - 1{\cdot}4x - 8{\cdot}51$ may be expressed in the form $(x + p)^2 - 9$, where p is a constant whose value is to be found.
Hence solve the quadratic equation $x^2 - 1{\cdot}4x - 8{\cdot}51 = 0$. [5]

2011 Summer

4. Express $-x^2 + 6x - 7$ in the form $-(x + a)^2 + b$, where the values of the constants a and b are to be found.
Hence sketch the graph of $y = -x^2 + 6x - 7$, indicating the coordinates of its stationary point. [4]

2012 Winter

5. *(a)* Express $3x^2 - 6x + 5$ in the form $a(x + b)^2 + c$, where a, b and c are constants whose values are to be found. [3]

(b) **Use your answer to part *(a)*** to find the greatest value of

$$\frac{1}{3x^2 - 6x + 11}.$$ [2]

2012 Summer

5. *(a)* Express $3x^2 - 12x + 29$ in the form $a(x + b)^2 + c$, where the values of the constants a, b and c are to be found. [3]

(b) **Using your answer to part *(a)***, write down the stationary value of $y = 3x^2 - 12x + 29$.
State whether this stationary value is a maximum or a minimum. [2]

2013 Winter

4. *(a)* (i) Express $x^2 + 8x + 5$ in the form $(x + a)^2 + b$, where the values of a, b are to be determined.

(ii) **Use your answers to part (i)** to find the least value of $3x^2 + 24x + 15$ and the corresponding value of x. [4]

(b) Solve the simultaneous equations $y = x^2 - x - 9$ and $y = 2x - 5$ algebraically. **Write down** a geometrical interpretation of your results. [5]

2013 Summer

4. *(a)* Express $2x^2 - 16x - 8$ in the form $a(x + b)^2 + c$, where the values of the constants a, b and c are to be found. [3]

(b) **Using your answer to part *(a)***, find the least value of $x^2 - 8x - 4$ and the corresponding value of x. [2]

2014 Winter

4. Show that $x^2 + 1{\cdot}6x - 24{\cdot}36$ may be expressed in the form $(x + p)^2 - 25$, where p is a constant whose value is to be found.
Hence solve the quadratic equation $x^2 + 1{\cdot}6x - 24{\cdot}36 = 0$. [5]

2014 Summer

5. *(a)* Express $4x^2 - 8x + 11$ in the form $a(x + b)^2 + c$, where a, b and c are constants whose values are to be found. [3]

(b) **Use your answer to part *(a)*** to find the greatest value of $\dfrac{1}{4x^2 - 8x + 29}$. [2]

2015

4. *(a)* Express $4x^2 - 24x - 189$ in the form $a(x + b)^2 + c$, where the values of the constants a, b and c are to be found. [3]

(b) **Using your answer to part *(a)***, solve the equation

$$4x^2 - 24x - 189 = 0.$$ [3]

2016

5. *(a)* Express $x^2 + 4x - 8$ in the form $(x + a)^2 + b$, where a and b are constants whose values are to be found. [2]

(b) Use an algebraic method to solve the simultaneous equations $y = x^2 + 4x - 8$ and $y = 2x + 7$. [4]

(c) Draw a sketch illustrating geometrically the results of both part *(a)* and part *(b)*. [4]

2017

4. *(a)* Express $-2x^2 - 20x + 35$ in the form $a(x + b)^2 + c$, where the values of the constants a, b and c are to be found. [3]

(b) **Without carrying out any further calculation**, write down the stationary value of $y = -2x^2 - 20x + 35$ and state whether this stationary value is a maximum or a minimum. [2]