C1 Differentiation Answers

Specimen

8. *(a)* $y+\delta y=(x+\delta x)^2-4(x+\delta x)+2.$ (o.e.) B1

$\delta y=(x+\delta x)^2-4(x+\delta x)+2$ (o.e.) M1

$-x^2+4x-2$

$=2x\delta x+(\delta x)^2-4\delta x.$ (o.e.) A1

$\therefore \frac{dy}{dx}=\lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\lim_{\delta x\to 0}(2x+\delta x-4)$ (o.e.) M1

$=2x-4.$ A1

(b) $3(-4)x^{-5}+4\cdot\frac{1}{2}x^{-\frac{1}{2}}$ M1 (attempted differentiation of x^{-4}
A1

$= \quad -\frac{12}{x^5}+\frac{2}{\sqrt{x}}.$ M1 (attempted differentiation of $x^{\frac{1}{2}}$)
A1

2005 Winter

(7) $y = x^2+4x+3$

$\frac{dy}{dx} = \lim_{\delta x\to 0}\frac{f(x+\delta x)-f(x)}{\delta x}$

$= \lim_{\delta x\to 0}\frac{[(x+\delta x)^2+4(x+\delta x)+3]-[x^2+4x+3]}{\delta x}$

$= \lim_{\delta x\to 0}\frac{[(x+\delta x)(x+\delta x)+4x+4\delta x+3]-x^2-4x-3}{\delta x}$

$= \lim_{\delta x\to 0}\frac{[x^2+x\delta x+x\delta x+(\delta x)^2+4x+4\delta x+3]-x^2-4x-3}{\delta x}$

$= \lim_{\delta x\to 0}\frac{2x\delta x+(\delta x)^2+4\delta x}{\delta x}$

$= \lim_{\delta x\to 0} 2x+\delta x+4$

$\frac{dy}{dx} = 2x+4.$

2005 Summer

5.	$y + \Delta y = (x + \Delta x)^2 - 7(x + \Delta x) + 2$	B1
	$\Delta y = (x + \Delta x)^2 - 7(x + \Delta x) + 2 - (x^2 - 7x + 2)$	M1
	$= 2x\Delta x + (\Delta x)^2 - 7\Delta x$	A1
	$\frac{\Delta y}{\Delta x} = 2x + \Delta x - 7$	M1 (divide by Δx and let $\Delta x \to 0$. Method must involve $\Delta x \to 0$ and some statement about answer being a limit)
	$\frac{dy}{dx} = \underset{\Delta x \to 0}{Lt} \frac{\Delta y}{\Delta x}$	
	$= 2x - 7$	A1 (award for clear presentation and no abuse of notation)
		[5]

2006 Winter

8.	*(a)*	Let $y = 2x^2 - 5x + 3$	
		$y + \Delta y = 2(x + \Delta x)^2 - 5(x + \Delta x) + 3$	B1
		$\Delta y = 4x\Delta x + 2(\Delta x)^2 - 5$	M1 (attempt to find Δy)
			A1
		$\frac{\Delta y}{\Delta x} = 4x + 2\Delta x - 5$	M1 (divide by Δx and let $\Delta x \to$ o)
		Let $\Delta x \to$ o	
		$\frac{dy}{dx} = \underset{\Delta x \to 0}{Lt} \frac{\Delta y}{\Delta x} = 4x - 5$	A1 (award only if some mention of limit, clear presentation and no abuse of notation)

(b)	$\frac{dy}{dx} = -\frac{a}{x^2} + 3x^{\frac{1}{2}}$	(o.e.)	B1, B1
	$-\frac{a}{4^2} + 3 \times 4^{\frac{1}{2}} = 7$		M1 (f' (4)=7, reasonable diffn)
	$a = -16$		A1 (C.A.O.)
			9

2006 Summer

7. (*a*) $y + \delta y = (x + \delta x)^2 - 3(x + \delta x) + 4$ B1

Subtracting y from above to find δy M1

$\delta y = 2x\delta x + (\delta x)^2 - 3\delta x$ A1

Dividing by δx, letting $\delta x \to 0$ and referring to limiting value of $\frac{\delta y}{\delta x}$ M1

$\frac{dy}{dx} = 2x - 3$ A1

(*b*) Required derivative $= -4x^{-3} + \frac{7x^{-\frac{1}{2}}}{2}$ B1, B1

2007 Winter

5. (*a*) $y = 2x^2 - 5x + 3$

$y + \delta y = 2(x + \delta x)^2 - 5(x + \delta x) + 3$ B1

Subtracting y from above to find δy M1

$\delta y = 4x\delta x + 2(\delta x)^2 - 5\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = 4x - 5$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = 3$ at $x = 2$ B1

Gradient of normal $= \frac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ M1

Equation of normal: $y - 1 = -⅓(x - 2)$ (or equivalent) A1

(f.t. candidate's numerical value for $\frac{dy}{dx}$)

2007 Summer

6. $y = x^2 - 12x + 10$

$y + \delta y = (x + \delta x)^2 - 12(x + \delta x) + 10$ B1

Subtracting y from above to find δy M1

$\delta y = 2x\delta x + (\delta x)^2 - 12\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = 2x - 12$ (c.a.o.) A1

2008 Winter

6. (*a*) $y + \delta y = 3(x + \delta x)^2 - 4(x + \delta x) + 7$ — B1

Subtracting y from above to find δy — M1

$\delta y = 6x\delta x + 3(\delta x)^2 - 4\delta x$ — A1

Dividing by δx and letting $\delta x \to 0$ — M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 6x - 4$ (c.a.o.) — A1

(*b*) Required derivative $= 5 \times ½ \times x^{-1/2} - 3 \times (-3) \times x^{-4}$ — B1, B1

2008 Summer

4. (*a*) $y + \delta y = 5(x + \delta x)^2 + 3(x + \delta x) - 4$ — B1

Subtracting y from above to find δy — M1

$\delta y = 10x\delta x + 5(\delta x)^2 + 3\delta x$ — A1

Dividing by δx and letting $\delta x \to 0$ — M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 10x + 3$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = -8 \times x^{-2} + 3 \times \frac{1}{2} \times x^{-1/2}$ — B1, B1

Either $4^{-2} = \frac{1}{16}$ **or** $4^{-1/2} = \frac{1}{2}$ — B1

$\frac{dy}{dx} = \frac{1}{4}$ (c.a.o) B1

2009 Winter

8. (*a*) $y + \delta y = 7(x + \delta x)^2 + 5(x + \delta x) - 2$ — B1

Subtracting y from above to find δy — M1

$\delta y = 14x\delta x + 7(\delta x)^2 + 5\delta x$ — A1

Dividing by δx and letting $\delta x \to 0$ — M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 14x + 5$ (c.a.o.) A1

(*b*) Required derivative $= 2 \times (-3) \times x^{-4} + 5 \times (^2/_3) \times x^{-1/3}$ — B1, B1

2009 Summer

5. (*a*) $y + \delta y = 4(x + \delta x)^2 - 5(x + \delta x) - 3$ B1

Subtracting y from above to find δy M1

$\delta y = 8x\delta x + 4(\delta x)^2 - 5\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 8x - 5$ (c.a.o.) A1

(*b*) Required derivative $= 7 \times {}^3/_4 \times x^{-1/4} - 2 \times (-4) \times x^{-5}$ B1, B1

2010 Winter

6. (*a*) $y + \delta y = 3(x + \delta x)^2 - 7(x + \delta x) - 5$ B1

Subtracting y from above to find δy M1

$\delta y = 6x\delta x + 3(\delta x)^2 - 7\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 6x - 7$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = a \times \frac{5}{2} \times x^{3/2}$ B1

Substituting $x = 4$ in candidate's expression for $\frac{dy}{dx}$ and putting

expression equal to -2 M1

$a = -\frac{1}{10}$ (c.a.o.) A1

2010 Summer

7. (*a*) $y + \delta y = -(x + \delta x)^2 + 5(x + \delta x) - 9$ B1

Subtracting y from above to find δy M1

$\delta y = -2x\delta x - (\delta x)^2 + 5\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = -2x + 5$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = \frac{3}{4} \times \frac{1}{3} \times x^{-2/3} + (-2) \times 12 \times x^{-3}$ B1, B1

Either $8^{-2/3} = \frac{1}{4}$ **or** second term $= (-)\frac{24}{512}$ (or equivalent fraction) B1

$\frac{dy}{dx} = \frac{1}{64}$ (or equivalent) (c.a.o.) B1

2011 Winter

4. (*a*) $y + \delta y = 6(x + \delta x)^2 + 4(x + \delta x) - 9$ B1

Subtracting y from above to find δy M1

$\delta y = 12x\delta x + 6(\delta x)^2 + 4\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 12x + 4$ (c.a.o.) A1

(*b*) Required derivative = $3 \times (-4) \times x^{-5} - 7 \times ({}^1/_3) \times x^{-2/3}$ B1, B1

2011 Summer

6. (*a*) $y + \delta y = 7(x + \delta x)^2 - 5(x + \delta x) + 2$ B1

Subtracting y from above to find δy M1

$\delta y = 14x\delta x + 7(\delta x)^2 - 5\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 14x - 5$ (c.a.o.) A1

(*b*) Required derivative = $4 \times {}^2/_5 \times x^{-3/5} - 9 \times (-1) \times x^{-2}$

(completely correct answer) B2

(one correct term) B1

2012 Winter

7.	(*a*)	$y + \delta y = 8(x + \delta x)^2 - 5(x + \delta x) - 6$	B1
		Subtracting y from above to find δy	M1
		$\delta y = 16x\delta x + 8(\delta x)^2 - 5\delta x$	A1
		Dividing by δx and letting $\delta x \to 0$	M1
		$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 16x - 5$	(c.a.o.) A1
	(*b*)	$\frac{dy}{dx} = a \times (-1) \times x^{-2} + 10 \times \frac{1}{2} \times x^{-1/2}$	B1, B1
		Attempting to substitute $x = 4$ in candidate's expression for $\frac{dy}{dx}$ and putting expression equal to 3	M1
		$a = -8$	(c.a.o.) A1

2012 Summer

7.	(*a*)	$y + \delta y = 3(x + \delta x)^2 - 7(x + \delta x) + 5$	B1
		Subtracting y from above to find δy	M1
		$\delta y = 6x\delta x + 3(\delta x)^2 - 7\delta x$	A1
		Dividing by δx and letting $\delta x \to 0$	M1
		$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 6x - 7$	(c.a.o.) A1
	(*b*)	Required derivative $= {}^{2}/_{3} \times {}^{1}/_{4} \times x^{-3/4} + 12 \times (-3) \times x^{-4}$	B1, B1

2013 Winter

6.	(*a*)	$y + \delta y = -(x + \delta x)^2 + 4(x + \delta x) - 6$	B1
		Subtracting y from above to find δy	M1
		$\delta y = -2x\delta x - (\delta x)^2 + 4\delta x$	A1
		Dividing by δx and letting $\delta x \to 0$	M1
		$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = -2x + 4$	(c.a.o.) A1
	(*b*)	$\frac{dy}{dx} = 5 \times \frac{4}{3} \times x^{1/3} - 9 \times -\frac{1}{2} \times x^{-3/2}$	B1, B1

2013 Summer

7.	(*a*)	$y + \delta y = 5(x + \delta x)^2 + 8(x + \delta x) - 11$	B1
		Subtracting y from above to find δy	M1
		$\delta y = 10x\delta x + 5(\delta x)^2 + 8\delta x$	A1
		Dividing by δx and letting $\delta x \to 0$	M1
		$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 10x + 8$	(c.a.o.) A1
	(*b*)	$\frac{dy}{dx} = 6 \times \frac{2}{3} \times x^{-1/3} + 5 \times -2 \times x^{-3}$ (completely correct answer)	B2
		(**If B2 not awarded**, award B1 for at least one correct non-zero term)	

2014 Winter

8.	**Either:** showing that $f(2) = 0$	
	Or: trying to find $f(r)$ for at least two values of r	M1
	$f(2) = 0 \Rightarrow x - 2$ is a factor	A1
	$f(x) = (x - 2)(6x^2 + ax + b)$ with one of a, b correct	M1
	$f(x) = (x - 2)(6x^2 - x - 2)$	A1
	$f(x) = (x - 2)(3x - 2)(2x + 1)$ (f.t. only $6x^2 + x - 2$ in above line)	A1
	$x = 2, \frac{2}{3}, -\frac{1}{2}$ (f.t. for factors $3x \pm 2$, $2x \pm 1$)	A1

Special case

Candidates who, after having found $x - 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1 for final 4 marks

2014 Summer

7.	(*a*)	$y + \delta y = -3(x + \delta x)^2 + 8(x + \delta x) - 7$	B1
		Subtracting y from above to find δy	M1
		$\delta y = -6x\delta x - 3(\delta x)^2 + 8\delta x$	A1
		Dividing by δx and letting $\delta x \to 0$	M1
		$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = -6x + 8$	(c.a.o.) A1
	(*b*)	$\frac{dy}{dx} = 9 \times \frac{5}{4} \times x^{1/4} - 8 \times -\frac{1}{3} \times x^{-4/3}$	B1, B1

2015

7. (*a*) $y + \delta y = 9(x + \delta x)^2 - 8(x + \delta x) - 3$ B1

Subtracting y from above to find δy M1

$\delta y = 18x\delta x + 9(\delta x)^2 - 8\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 18x - 8$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = 3 \times (-6) \times x^{-7} - 4 \times \frac{5}{3} \times x^{2/3}$ B1 B1

2016

8. (*a*) $y + \delta y = 10(x + \delta x)^2 - 7(x + \delta x) - 13$ B1

Subtracting y from above to find δy M1

$\delta y = 20x\delta x + 10(\delta x)^2 - 7\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 20x - 7$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = 4 \times \frac{1}{2} \times x^{-1/2} + (-1) \times 45 \times x^{-2}$ B1, B1

Either $9^{-1/2} = \frac{1}{3}$ **or** $9^{-2} = \frac{1}{81}$ (or equivalent fraction) B1

$\frac{dy}{dx} = \frac{1}{9}$ (or equivalent) (c.a.o.) B1

2017

9. (*a*) $y + \delta y = -5(x + \delta x)^2 - 7(x + \delta x) + 13$ B1

Subtracting y from above to find δy M1

$\delta y = -10x\delta x - 5(\delta x)^2 - 7\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = -10x - 7$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = 6 \times \frac{3}{4} \times x^{-1/4} + 5 \times -3 \times x^{-4}$ (completely correct answer) B2

(**If B2 not awarded**, award B1 for at least one correct non-zero term)