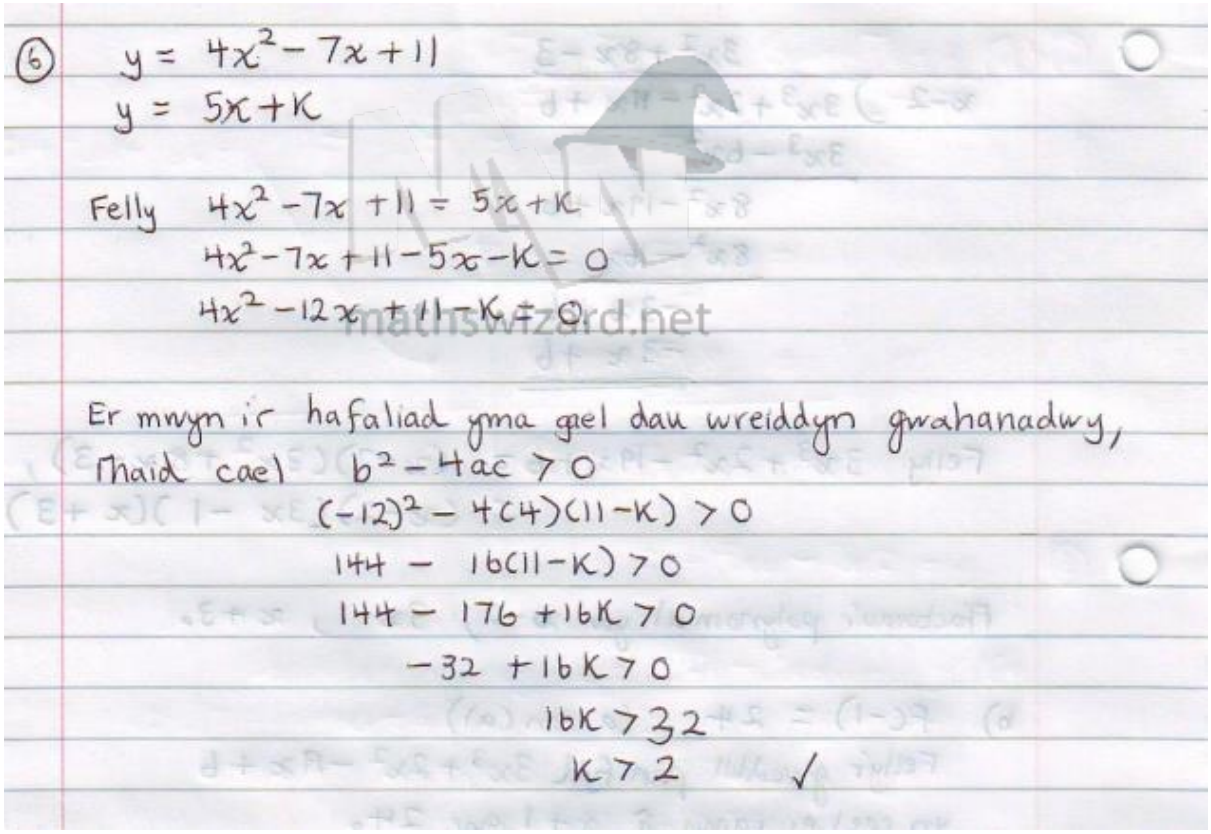
C2 Roots of Equation Answers

Specimen

4.	Condition for distinct real roots is		M1 (Use of $b^2 - 4ac$)
		$16 - 4 \times 2k(k-1) > 0$	A1
	$\therefore$	$2 - k^2 + k > 0$	
		$k^2 - k - 2 < 0$	A1 (convincing)
	giving	$(k-2)(k+1) < 0$	B1 (fixed points)
	then	$-1 < k < 2$ or $(-1,2)$.	M1 (any method) A1

2005 Winter



2005 Summer

7. (a) $\left(\frac{dy}{dx} = 0\right)$

$3x^2 - 6x = 0$ — B1 $\left(\frac{dy}{dx}\right)$; M1 $\left(\frac{dy}{dx} = 0\right)$

$3x(x-2) = 0$

$x = 0, 2$ — A1 (either root)

When $x = 0$, $y = 0$; when $x = 2$, $y = -4$ — A1 (both, C.A.O.)

$\frac{d^2y}{dx^2} = 6x - 6$ — M1 (any method)

$x = 0$, $\frac{d^2y}{dx^2} = -6 < 0$ max. pt — A1

$x = 2$, $\frac{d^2y}{dx^2} = 6 > 0$ min. pt — A1

2006 Winter

5. For no real roots, — M1 (b^2-4ac, correct b, a or c correct)

$4^2 - 4(k+2)(k+5) < 0$ — A1 (correct)
$4 - k^2 - 7k - 10 < 0$ — M1 ($b^2 - 4ac < 0$)
$k^2 + 7k + 6 > 0$ — A1 (convincing)

$(k+6)(k+1) > 0$ — B1 (fixed pts, –1, –6)
$k < -6$ (and/or) $k > -1$

or — B2 (F.T. fixed points)

$k < -6, k > -1$

$-1 < k < -6$ or $k > -1$ or $k < -6$ — All gain B1

7

2006 Summer

4. (*a*) An expression for $b^2 - 4ac$, with $b = \pm 4$, and at least one of a or c correct — M1

$b^2 - 4ac = 4^2 - 4k(k-3)$ — A1
$b^2 - 4ac = 4(k-4)(k+1)$ — A1
Putting $b^2 - 4ac = 0$ — m1
$k = -1, 4$ (f.t. one slip) — A1

(*b*) $a = 4$ — B1
$b = -14$ — B1
Least value $= -14$ (f.t. candidate's b) — B1

2007 Winter

7. An expression for $b^2 - 4ac$, with $b = \pm 4$, and at least one of a or c correct — M1

$b^2 - 4ac = (\pm 4)^2 - 4k(k-3)$ — A1
Putting $b^2 - 4ac \geq 0$ or $b^2 - 4ac > 0$ — m1
$k^2 - 3k - 4 \leq 0$ (convincing) — A1
Finding fixed points $k = 4$, $k = -1$ (c.a.o.) — B1
$-1 \leq k \leq 4$ **or** $4 \geq k \geq -1$ **or** $[-1, 4]$ **or** a correctly worded statement to the effect that k lies between -1 and 4 inclusive
(f.t. candidate's fixed points) — B2

Note: $-1 < k < 4$,
$k \leq 4, -1 \leq k$
$k \leq 4$ or $-1 \leq k$
all earn B1

2007 Summer

8. (*a*) An expression for $b^2 - 4ac$, with $b = (2k+1)$, and at least one of a or c correct — M1

$b^2 - 4ac = (2k+1)^2 - 4(1)(k^2 + k + 1)$ — A1
$b^2 - 4ac = -3$ (or < 0, convincing) — A1
$b^2 - 4ac < 0 \Rightarrow$ no real roots (f.t. one slip) — A1

(*b*) Finding critical points $x = -3$, $x = -\frac{1}{2}$ — B1
$-3 < x < -\frac{1}{2}$ **or** $-\frac{1}{2} > x > -3$ **or** $(-3, -\frac{1}{2})$ **or** $-3 < x$ and $x < -\frac{1}{2}$
or a correctly worded statement to the effect that x lies strictly between -3 and $-\frac{1}{2}$
(f.t. candidate's critical points) — B2

Note: $-3 \leq x \leq -\frac{1}{2}$,
$-3 < x, x < -\frac{1}{2}$
$-3 < x \;\; x < -\frac{1}{2}$
$-3 < x$ or $x < -\frac{1}{2}$
all earn B1

2008 Winter

5. (*a*) An expression for $b^2 - 4ac$, with $c = \pm k$ and at least one of a or b correct M1

$b^2 - 4ac = 2^2 - 4 \times 3 \times (-k)$ A1

Putting $b^2 - 4ac > 0$ m1

$4 + 12k > 0 \Rightarrow k > -{}^1/_3$ (o.e.)

(f.t. only for $c = k$ in original expression for $b^2 - 4ac$) A1

(*b*) Finding critical points $x = -2$, $x = 7$ B1

$-2 \leq x \leq 7$ **or** $7 \geq x \geq -2$ **or** $[-2, 7]$ **or** $x \leq 7$ and $-2 \leq x$ **or** a correctly worded statement to the effect that x lies between –2 and 7 (inclusive)

(f.t. candidate's critical points) B2

Note: $-2 < x < 7$,
$x \leq 7, -2 \leq x$
$x \leq 7 \quad -2 \leq x$
$x \leq 7$ or $-2 \leq x$ all earn B1

2008 Summer

10. (*a*) Finding critical points $x = -1{\cdot}5$, $x = 3$ B1

A statement (mathematical or otherwise) to the effect that $x \leq -1{\cdot}5$ or $3 \leq x$ (or equivalent)

(f.t. candidate's critical points) B2

Deduct 1 mark for each of the following errors
the use of strict inequalities
the use of the word 'and' instead of the word 'or'

(*b*) (i) An expression for $b^2 - 4ac$, with $b = (-)6$ and at least one of a or c correct M1

$b^2 - 4ac = [(-)6]^2 - 4 \times 3 \times m$ A1

$b^2 - 4ac < 0$ m1

$m > 3$ (c.a.o.) A1

(ii) $3x^2 - 4x + 7 = 2x + k$ M1

No points of intersection $\Leftrightarrow 3x^2 - 6x + (7 - k) = 0$ has no real roots (allow one slip in quadratic) m1

$4 > k$ A1

(if candidate uses (*b*)(i), f.t. candidate's inequality for m)

2009 Winter

5. An expression for $b^2 - 4ac$, with at least two of a, b, c correct — M1

$b^2 - 4ac = 8^2 - 4 \times (3k - 2) \times k$ — A1

Putting $b^2 - 4ac < 0$ — m1

$3k^2 - 2k - 16 > 0$ (convincing) — A1

Finding critical points $k = -2$, $k = {}^8/_3$ — B1

A statement (mathematical or otherwise) to the effect that
$k < -2$ or ${}^8/_3 < k$ (or equivalent) (f.t. candidate's critical points) — B2

Deduct 1 mark for each of the following errors
the use of non-strict inequalities
the use of the word 'and' instead of the word 'or'

2009 Summer

6. (*a*) An expression for $b^2 - 4ac$, with at least two of a, b, c correct — M1

$b^2 - 4ac = (2k)^2 - 4(k+1)(k-1)$ — A1

$b^2 - 4ac = 4$ (c.a.o.) — A1

candidate's value for $b^2 - 4ac > 0$ ($\Rightarrow$ two distinct real roots) — A1

(*b*) Finding critical values $x = -2$, $x = {}^3/_5$ — B1

$-2 \le x \le {}^3/_5$ **or** ${}^3/_5 \ge x \ge -2$ **or** $[-2, {}^3/_5]$ **or** $-2 \le x$ and $x \le {}^3/_5$
or a correctly worded statement to the effect that x lies between -2 and ${}^3/_5$ (both inclusive)
(f.t. critical values ± 2, $\pm {}^3/_5$) — B2

Note: $-2 < x < {}^3/_5$,
$-2 \le x, x \le {}^3/_5$
$-2 \le x \ \ x \le {}^3/_5$
$-2 \le x$ or $x \le {}^3/_5$
all earn B1

2010 Winter

5. (*a*) An expression for $b^2 - 4ac$, with at least two of a, b or c correct M1
$b^2 - 4ac = 3^2 - 4 \times k \times (-5)$ A1
$b^2 - 4ac < 0$ m1
$k < -{}^{9}/_{20}$
(f.t. only for $k > {}^{9}/_{20}$ from $b^2 - 4ac = 3^2 - 4 \times k \times 5$) A1

(*b*) Finding critical values $x = -1{\cdot}5$, $x = 2$ B1
A statement (mathematical or otherwise) to the effect that
$x < -1{\cdot}5$ or $2 < x$ (or equivalent)
(f.t critical values $\pm 1{\cdot}5, \pm 2$ only) B2
Deduct 1 mark for each of the following errors
the use of $\leq$ rather than $<$
the use of the word 'and' instead of the word 'or'

2010 Summer

6. (*a*) An expression for $b^2 - 4ac$, with at least two of a, b or c correct M1
$b^2 - 4ac = k^2 - 4 \times 2 \times 18$ A1
Candidate's expression for $b^2 - 4ac < 0$ m1
$-12 < k < 12$ (c.a.o.) A1

(*b*) Finding critical values $x = -0{\cdot}5$, $x = 0{\cdot}6$ B1
A statement (mathematical or otherwise) to the effect that
$x \leq -0{\cdot}5$ or $0{\cdot}6 \leq x$ (or equivalent) (f.t. only $x = \pm 0{\cdot}5, x = \pm 0{\cdot}6$) B2
Deduct 1 mark for each of the following errors
the use of $<$ rather than $\leq$
the use of the word 'and' instead of the word 'or'

2011 Winter

3. An expression for $b^2 - 4ac$, with at least two of a, b, c correct — M1

$b^2 - 4ac = (3k - 1)^2 - 4 \times 2 \times (3k^2 - 1)$ — A1

Putting $b^2 - 4ac > 0$ — m1

$5k^2 + 2k - 3 < 0$ (convincing) — A1

Finding critical values $k = -1$, $k = {}^3/_5$ — B1

$-1 < k < {}^3/_5$ **or** ${}^3/_5 > k > -1$ **or** $(-1, {}^3/_5)$ **or** $-1 < k$ and $k < {}^3/_5$ **or** a correctly worded statement to the effect that k lies strictly between -1 and ${}^3/_5$

(f.t. only critical values of ± 1 and $\pm {}^3/_5$) — B2

Note:

$-1 \leq k \leq {}^3/_5$

$-1 < k, k < {}^3/_5$

$-1 < k \;\; k < {}^3/_5$

$-1 < k$ or $k < {}^3/_5$

all earn B1

2011 Summer

5. (*a*) $x^2 + (4k + 3)x + 7 = x + k$ — M1

$x^2 + (4k + 2)x + (7 - k) = 0$ — A1

An attempt to apply $b^2 - 4ac$ to the candidate's quadratic — M1

$b^2 - 4ac = (4k + 2)^2 - 4 \times 1 \times (7 - k)$

(f.t. candidate's quadratic) — A1

Candidate's expression for $b^2 - 4ac > (\geq) 0$ — m1

$4k^2 + 5k - 6 > 0$ (convincing) — A1

(*b*) Finding critical values $k = -2$, $k = 0{\cdot}75$ — B1

A statement (mathematical or otherwise) to the effect that $k < -2$ or $0{\cdot}75 < k$ (or equivalent) (f.t. only $k = \pm 2$, $k = \pm 0{\cdot}75$) — B2

Deduct 1 mark for each of the following errors

the use of $\leq$ rather than $<$

the use of the word 'and' instead of the word 'or'

2012 Winter

6.	An expression for $b^2 - 4ac$, with at least two of a, b, c correct	M1
	$b^2 - 4ac = 4^2 - 4 \times (k+6) \times (k+3)$	A1
	Putting $b^2 - 4ac < 0$	m1
	$k^2 + 9k + 14 > 0$ (convincing)	A1
	Finding critical values $k = -7$, $k = -2$	B1
	A statement (mathematical or otherwise) to the effect that $k < -7$ or $-2 < k$ (or equivalent) (f.t. only critical values of ± 7 and ± 2)	B2
	Deduct 1 mark for each of the following errors: the use of non-strict inequalities the use of the word 'and' instead of the word 'or'	

2012 Summer

6.	(*a*)	An expression for $b^2 - 4ac$, with at least two of a, b, c correct	M1
		$b^2 - 4ac = (2k-1)^2 - 4(k^2 - k + 2)$	A1
		$b^2 - 4ac = -7$ (c.a.o.)	A1
		candidate's value for $b^2 - 4ac < 0$ ($\Rightarrow$ no real roots)	A1
	(*b*)	Finding critical values $x = -6$, $x = {}^2/_3$	B1
		A statement (mathematical or otherwise) to the effect that $x < -6$ or ${}^2/_3 < x$ (or equivalent) (f.t critical values ± 6, $\pm {}^2/_3$ only)	B2
		Deduct 1 mark for each of the following errors the use of $\leq$ rather than $<$ the use of the word 'and' instead of the word 'or'	

2013 Winter

5. (*a*) An expression for $b^2 - 4ac$, with at least two of a, b or c correct M1

$b^2 - 4ac = 6^2 - 4 \times 5 \times (-3k)$ A1

$b^2 - 4ac > 0$ m1

$k > -{}^3/_5$ (o.e.)

[f.t. only for $k < {}^3/_5$ from $b^2 - 4ac = 6^2 - 4 \times 5 \times (3k)$] A1

(*b*) Finding critical values $x = 2{\cdot}5$, $x = 3$ B1

$2{\cdot}5 \leq x \leq 3$ **or** $3 \geq x \geq 2{\cdot}5$ **or** $[2{\cdot}5, 3]$ **or** $2{\cdot}5 \leq x$ and $x \leq 3$ **or** a correctly worded statement to the effect that x lies between 2·5 and 3 (both values inclusive) (f.t. candidate's derived critical values) B2

Note:

$2{\cdot}5 < x < 3$

$2{\cdot}5 \leq x, x \leq 3$

$2{\cdot}5 \leq x \ \ x \leq 3$

$2{\cdot}5 \leq x$ or $x \leq 3$

all earn B1

2013 Summer

6. (*a*) (i) An expression for $b^2 - 4ac$, with at least two of a, b, c correct M1

$b^2 - 4ac = (4k + 1)^2 - 4 \times (k + 1) \times (k - 5)$ A1

Putting $b^2 - 4ac = 0$ m1

$4k^2 + 8k + 7 = 0$ (convincing) A1

(ii) An expression for $b^2 - 4ac$, with at least two of a, b, c correct (M1)

(to be awarded only if corresponding M1 is not awarded in part (i))

$b^2 - 4ac = 64 - 112$ $(= -48)$ A1

$b^2 - 4ac < 0 \Rightarrow$ no real roots A1

Note: Total mark for part (*a*) is 6 marks

(*b*) Finding critical values $x = -{}^3/_4$, $x = 3$ B1

A statement (mathematical or otherwise) to the effect that

$x \leq -{}^3/_4$ or $3 \leq x$ (or equivalent)

(f.t. candidate's derived critical values) B2

Deduct 1 mark for each of the following errors

the use of strict inequalities

the use of the word 'and' instead of the word 'or'

2014 Winter

6.	An expression for $b^2 - 4ac$, with at least two of a, b, c correct		M1
	$b^2 - 4ac = 8^2 - 4 \times (2k - 3) \times (2k + 3)$		A1
	Putting $b^2 - 4ac <(\leq) 0$		m1
	$100 - 16k^2 < 0$ (o.e.)	(c.a.o.)	A1
	Finding critical values $k = -5/2$, $k = 5/2$		
	(o.e.)	(f.t. candidate's values for m, n)	B1
	$k < -5/2$ or $5/2 < k$ (o.e.)	(f.t. only critical values of $-a$ and a)	B1
	Each of the following errors earns a final B0		
	the use of non-strict inequalities		
	the use of the word 'and' instead of the word 'or'		

2014 Summer

6.	An expression for $b^2 - 4ac$, with at least two of a, b, c correct		M1
	$b^2 - 4ac = (2k)^2 - 4 \times (k - 1) \times (7k - 4)$		A1
	Putting $b^2 - 4ac < 0$		m1
	$6k^2 - 11k + 4 > 0$	(convincing)	A1
	Finding critical values $k = {}^1/_2$, $k = {}^4/_3$		B1
	A statement (mathematical or otherwise) to the effect that		
	$k < {}^1/_2$ or $k > {}^4/_3$ (or equivalent)		
	(f.t. candidate's derived critical values)		B2
	Deduct 1 mark for each of the following errors		
	the use of non-strict inequalities		
	the use of the word 'and' instead of the word 'or'		

2015

5.	(*a*)	An expression for $b^2 - 4ac$, with at least two of a, b or c correct		M1
		$b^2 - 4ac = (2k - 5)^2 - 4 \times k \times (k - 6)$		A1
		Putting $b^2 - 4ac < 0$		m1
		$k < -\frac{25}{4}$	(or equivalent)	A1
	(*b*)	$k = -\frac{25}{4}$	[f.t. the end point(s) of the candidate's range in (*a*)]	B1

2016

6. (*a*) An expression for $b^2 - 4ac$, with at least two of a, b or c correct M1

$b^2 - 4ac = 8^2 - 4 \times 9 \times (-2k)$ A1

$b^2 - 4ac > 0$ m1

$k > -{}^{8}/_{9}$ (o.e.)

[f.t. only for $k < {}^{8}/_{9}$ from $b^2 - 4ac = 8^2 - 4 \times 9 \times (2k)$] A1

(*b*) Attempting to rewrite the inequality in the form $5x^2 - 7x - 6 \geq 0$ and an attempt to find the critical values M1

Critical values $x = -0{\cdot}6$, $x = 2$ A1

A statement (mathematical or otherwise) to the effect that

$x \leq -0{\cdot}6$ or $2 \leq x$ (or equivalent)

(f.t. candidate's derived critical values) A2

Deduct 1 mark for each of the following errors

the use of strict inequalities

the use of the word 'and' instead of the word 'or'

2017

(*c*) Use of both $k = -15$, $k = 17$ to find the range of values for k

(f.t. candidate's y-values at stationary points) M1

$k < -15$ or $17 < k$ (f.t. candidate's y-values at stationary points) A1