

## C1 Stationary Points Questions

Specimen

**10.** The curve  $C$  has equation

$$y = x^3 - 3x^2 - 9x + 3.$$

- (a) Find the coordinates and nature of the stationary points of  $C$ . [8]
- (b) Sketch  $C$ . [3]

2005 Winter

**3.** A curve  $C$  has equation

$$y = 2x^3 - 6x^2 + 12.$$

- (a) Find the coordinates of the stationary points of  $C$  and determine the nature of each of those points. [7]
- (b) Sketch  $C$ . [3]
- (c) State, giving a reason, the number of real roots of the equation  $2x^3 - 6x^2 + 12 = 0$ . [2]

2005 Summer

**7.** The curve  $C$  has equation  $y = x^3 - 3x^2$ .

- (a) Find the coordinates of the stationary points of  $C$  and determine the nature of each of these points. [7]
- (b) Sketch  $C$ . [3]
- (c) Find the range of values of  $k$  for which there are three real and distinct solutions of the equation  $x^3 - 3x^2 = k$ . [2]

2006 Winter

10. The curve  $C$  has equation

$$y = 2 + 6x^2 - 2x^3.$$

(a) Find the coordinates and the nature of each of the stationary points of  $C$ . [7]

(b) Sketch  $C$ , indicating clearly the nature of each of the stationary points. [3]

(c) State clearly, giving a reason, the number of real roots of the equation

$$2 + 6x^2 - 2x^3 = 0. \quad [2]$$

2006 Summer

10. The curve  $C$  has equation

$$y = x^3 - 3x^2 - 9x + 2.$$

Find the coordinates of the stationary points of  $C$  and determine the nature of each of these stationary points. [7]

2007 Winter

9. The curve  $C$  has equation

$$y = 4x^3 - 12x + 3.$$

(a) Find the coordinates of the stationary points of  $C$  and determine the nature of each of these points. [7]

(b) Sketch  $C$ , indicating the coordinates of the stationary points. [3]

(c) Given that  $f(x) = 4x^3 - 12x + 3$ , sketch the curve  $y = f(x - 1)$ , indicating the coordinates of **each** of the stationary points. [3]

2007 Summer

10. The curve  $C$  has equation

$$y = x^3 - x^2 - x + 2.$$

Find the coordinates of the stationary points of  $C$  and determine the nature of each of these stationary points. [7]

2008 Winter

10. The curve  $C$  has equation

$$y = x^3 - 12x + 11.$$

(a) Find the coordinates and nature of each of the stationary points of  $C$ . [7]

(b) Sketch  $C$ , indicating the coordinates of each of the stationary points. [2]

(c) Given that the equation

$$x^3 - 12x + 11 = k$$

has only one real root, find the range of possible values for  $k$ . [2]

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2008 Summer

9. The curve  $C$  has equation

$$y = -2x^3 + 3x^2 + 12x - 5.$$

Find the coordinates and nature of each of the stationary points of  $C$ . [7]

2009 Winter

10. The curve  $C$  has equation

$$y = x^3 + 3x^2 - 9x - 13.$$

(a) Find the stationary points of  $C$  and determine the nature of each of these points. [6]

(b) Sketch  $C$ , indicating the coordinates of the stationary points. [2]

(c) State, giving a reason, the number of real roots of the equation

$$x^3 + 3x^2 - 9x - 13 = 0. [2]$$

2009 Summer

10. The curve  $C$  has equation

$$y = x^3 - 3x^2 + 3x + 5.$$

- (a) Show that  $C$  has only one stationary point. Find the coordinates of this point. [4]
- (b) Verify that this stationary point is a point of inflection. [2]

2010 Winter

10. The curve  $C$  has equation

$$y = x^3 - 6x^2 + 20.$$

- (a) Find the coordinates and the nature of each of the stationary points of  $C$ . [6]
- (b) Sketch  $C$ , indicating the coordinates of each of the stationary points. [2]
- (c) Given that the equation

$$x^3 - 6x^2 + 20 = k$$

has three **distinct** real roots, find the range of possible values for  $k$ . [2]

2010 Summer

10. The curve  $C$  has equation

$$y = \frac{1}{2}x^3 - 6x + 3.$$

Find the coordinates and the nature of each of the stationary points of  $C$ . [6]

2011 Winter

10. The curve  $C$  has equation

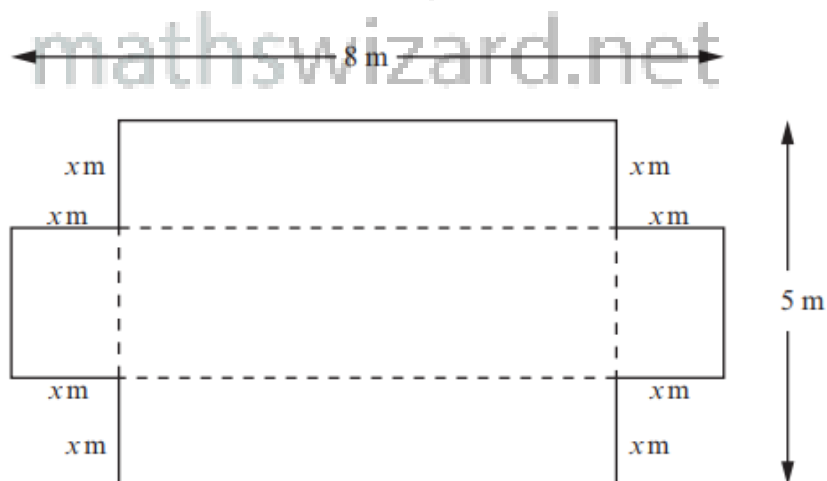
$$y = x^3 + kx^2 - 9x - 10,$$

where  $k$  is a constant. The two stationary points on the graph of  $C$  are denoted by  $Q$  and  $R$ . The  $x$ -coordinate of  $Q$  is  $-1$ .

- (a) Find  $\frac{dy}{dx}$  and hence show that  $k = -3$ . [3]
- (b) Find the  $x$ -coordinate of  $R$ . [2]
- (c) Determine the nature of each of the stationary points  $Q$  and  $R$ . [2]

2011 Summer

10. A rectangular sheet of metal has length 8 m and width 5 m. Four squares, each of side  $x$  m, where  $x < 2.5$ , have been cut away from the corners of the rectangular sheet, as shown in the diagram below. The rest of the metal sheet is now bent along the dotted lines to form an open tank in the form of a cuboid.



(a) Show that the volume  $V \text{ m}^3$  of this tank is given by

$$V = 4x^3 - 26x^2 + 40x. \quad [2]$$

(b) Find the maximum value of  $V$ , showing that the value you have found is a maximum value. [5]

2012 Winter

10. The curve  $C$  has equation

$$y = x^3 - 6x^2 + 12x - 9.$$

- (a) Show that  $C$  has only one stationary point. Find the coordinates of this point. [4]
- (b) Verify that this stationary point is a point of inflection. [2]

2012 Summer

10. The curve  $C$  has equation

$$y = x^3 + 3x^2 - 1.$$

- (a) Find the coordinates and the nature of each of the stationary points of  $C$ . [6]
- (b) Sketch  $C$ , indicating the coordinates of each of the stationary points. [2]
- (c) Write down the number of **positive** real roots of the equation

$$x^3 + 3x^2 - 1 = 0. \quad [1]$$

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2013 Winter

10. The curve  $C$  has equation

$$y = x^3 - 3x^2 - 9x + 14.$$

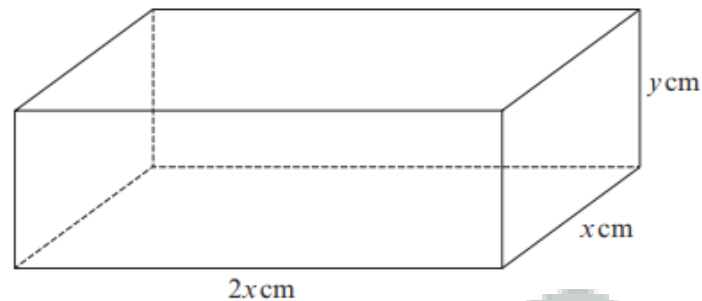
- (a) Find the coordinates and the nature of each of the stationary points of  $C$ . [6]
- (b) Sketch  $C$ , indicating the coordinates of each of the stationary points. [2]
- (c) Given that the equation

$$x^3 - 3x^2 - 9x + 14 = k$$

has only one real root, find the range of possible values for  $k$ . [2]

2013 Summer

10. The diagram shows a **closed** box in the form of a cuboid. The length of the box is  $2x$  cm, its width is  $x$  cm and its height is  $y$  cm.



The total surface area of the box is  $108 \text{ cm}^2$ .

- (a) (i) Write down an equation involving  $x$  and  $y$  and hence show that
- $$xy = 18 - \frac{2}{3}x^2.$$
- (ii) Hence show that the volume  $V \text{ cm}^3$  of the box is given by
- $$V = 36x - \frac{4}{3}x^3. \quad [3]$$
- (b) Find the maximum value of  $V$ , showing that the value you have found is a maximum value. [5]

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2014 Winter

10. The curve  $C$  has equation

$$y = -2x^3 + 12x^2 - 18x + 5.$$

- (a) Find the coordinates and the nature of each of the stationary points of  $C$ . [6]
- (b) Sketch  $C$ , indicating the coordinates of each of the stationary points. [2]
- (c) Given that the equation

$$-2x^3 + 12x^2 - 18x + 5 = k$$

- has three distinct real roots, find the range of possible values for  $k$ . [2]

2014 Summer

10. The curve  $C$  has equation

$$y = x^3 + 9x^2 + 27x + 31.$$

- (a) Show that  $C$  has only one stationary point. Find the coordinates of this point. [4]
- (b) Verify that this stationary point is a point of inflection. [2]
- (c) Sketch the graph of  $C$ , indicating the coordinates of its stationary point. [1]

END OF PAPER

2015

10. A sheep farmer wishes to construct a rectangular enclosure for his animals. He decides to use a straight wall as one side of the enclosure and fencing for the other three sides. The area of the enclosure is to be  $800\text{m}^2$ . The lengths of the sides of the rectangular enclosure are  $x\text{m}$  and  $y\text{m}$ , as shown in the diagram, and the total length of the fencing is  $L\text{m}$ .



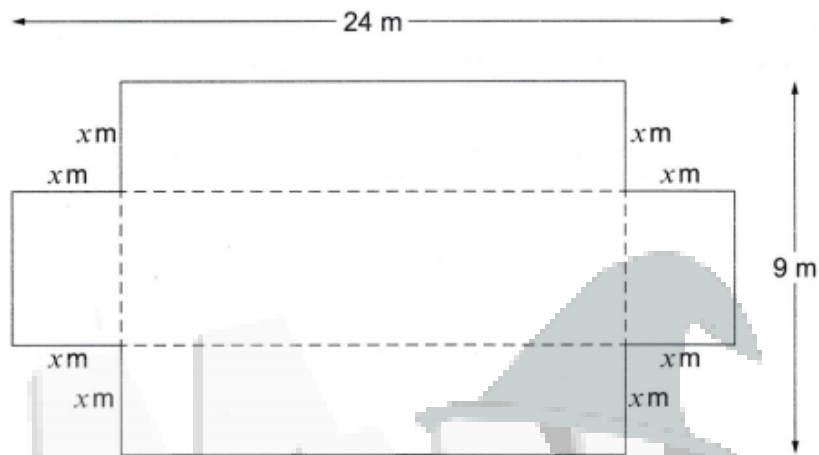
- (a) Show that  $L = x + \frac{1600}{x}$ . [2]
- (b) Find the minimum value of  $L$ , showing that the value you have found is a minimum value. [5]

END OF PAPER



2016

10. A rectangular sheet of metal has length 24 m and width 9 m. Four squares, each of side  $x$  m, where  $x < 4.5$ , have been cut away from the corners of the rectangular sheet, as shown in the diagram below. The rest of the metal sheet is now bent along the dotted lines to form an open tank in the form of a cuboid.



- (a) Show that the volume  $V \text{ m}^3$  of this tank is given by

$$V = 4x^3 - 66x^2 + 216x.$$

[2]

- (b) Find the maximum value of  $V$ , showing that the value you have found is a maximum value.

[5]

2017

10. The curve  $C$  has equation

$$y = x^3 - 9x^2 + 15x + 10.$$

- (a) (i) Find the coordinates of each of the stationary points of  $C$ .

- (ii) Determine the nature of each of these stationary points.

[6]

- (b) Sketch  $C$ , indicating the coordinates of each of the stationary points.

[2]

- (c) Given that the equation

$$x^3 - 9x^2 + 15x + 10 = k$$

has only one real root, find the range of possible values for  $k$ .

[2]