C2 Arithmetic Series Answers

Specimen

5. (*a*) $a + 3d = 11$ B1

$a + 5d = 17$ B1

$d = 3,\ a = 2$ M1(attempt to solve) A1

(*b*) $S_8 = \frac{8}{2}\,[2 \times 2 + 7 \times 3] = 100$ B1

2005 Winter

(5) $a + (a + 4d) = 0.$ —(1)

$a + 12d = 20$ —(2)

a) (2) $\Rightarrow$ $a = 20 - 12d$ —(3)

Yn amnewid o (3) i mewn i (1):

$20 - 12d + 20 - 12d + 4d = 0$

$40 - 20d = 0$

$-40 + 20d = 0$

$20d = 40$

$\underline{\underline{d = 2}}$

Felly $a = 20 - 12(2)$

$a = 20 - 24$

$\underline{\underline{a = -4}}$

b) $S_{20} = \frac{20}{2}\left(-8 + 19(2)\right)$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$= 10(-8 + 38)$

$= 10(30)$

$= 300.$

2005 Summer

3. (a) n^{th} term $= a + (n - 1)d$ (must be displayed)

$S_n = a + (a + d) + \ldots\ldots\ldots\ldots + a - (n - 2)d + a + (n - 1)d$

B1 (at least 3 terms, one at each end)

$S_n = a + (n - 1)d + a + (n - 2)d + \ldots\ldots\ldots\ldots\ldots\ldots + (a + d) + a$

M1 (reverse and add)

$2S_n = (2a + (n - 1)d) + (2a + (n - 1)d) + \ldots\ldots\ldots + (2a + (n - 1)d) + (2a + (n - 1)d)$

(n terms)

$= n\left[2a + (n-1)d\right]$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ A1 (convincing)

2006 Winter

5. *(a)* $2a + d = 3$ (1) B1 ($a + a + d = 3$)

$a + 7d = 47$ (2) B1

Solve (1), (2) $d = 7, a = -2$ M1 (attempt to solve)
A1 (C.A.O.)

(b) $S_{20} = \frac{20}{2}[2 \times -2 + 19 \times 7]$ M1 (correct formula with candidate values)

$= 1290$ A1 (F.T. candidate values)

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2006 Summer

4. *(a)* $S_n = a + a + d + \ldots + a + (n-2)d + a(n-1)d$ — B1 (at least 3 terms one at each end)

$S_n = a + (n-1)d + a + (n-2)d + \ldots + a + d + a$

$2S_n = 2a + (n-1)d + 2a + (n-1)d + \ldots$ — M1

$+ 2a + (n-1)d + 2a + (n-1)d$

$= n[2a + (n-1)d]$

$\therefore\ S_n = \frac{n}{2}[2a + (n-1)d]$ — A1 (convincing)

(b) (i) $\frac{20}{2}[2a + 19d] = 540$ — B1

$\frac{30}{2}[2a + 29d] = 1260$ — B1

$2a + 19d = 54$ (1)

$2a + 29d = 84$ (2)

Solve (1), (2), $d = 3$ — M1 (reasonable attempt to solve equations)

$a = -\frac{3}{2}$ — A1 (both) C.A.O.

(ii) 50th term $= -\frac{3}{2} + (n-1)3$ $(n = 50)$ — M1 (correct)

$= 145\cdot5$ — A1 (F.T. derived values)

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2007 Winter

4. $a + 7d = k(a + 2d)$ $(k = 2, ½)$ M1

$a + 7d = 2(a + 2d)$ A1

$a + 19d = 11$ B1

An attempt to solve simultaneous equations M1

$d = \frac{1}{2},\ a = \frac{3}{2}$ (both values needed) (f.t. only for $k = ½$) A1

2007 Summer

4. (*a*)

$a + 2d = k(a + 5d)$	$(k = 4, {}^1/_4)$	M1
$a + 2d = 4(a + 5d)$		A1
$\frac{20}{2}[2a + 19d] = 350$		B1
An attempt to solve simultaneous equations		M1
$d = 5$ $(a = -30)$	(c.a.o.)	A1
$a = -30$ $(d = 5)$	(f.t. one error)	A1

(*b*)

$-30 + (n - 1) \times 5 = 125$	(equation for n'th term and an attempt to solve, f.t. candidate's values for a, d)	M1
$n = 32$	(c.a.o.)	A1

2008 Winter

3. (*a*)

$S_n = a + [a + d] + \ldots + [a + (n - 2)d] + [a + (n - 1)d]$	(at least 3 terms, one at each end)	B1
$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + [a + d] + a$		
$2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \ldots + [a + a + (n - 1)d]$	(reverse and add)	M1
$2S_n = n[2a + (n - 1)d]$		
$S_n = \frac{n}{2}[2a + (n - 1)d]$	(convincing)	A1

(*b*)

$S_n = \frac{n}{2}[2 + (n - 1)2]$		B1
$S_n = n^2$	(c.a.o.)	B1

(*c*)

$a + 19d = 98$		B1
$\frac{20}{2} \times [2a + 19d] = 1010$		B1
An attempt to solve the above equations simultaneously by eliminating one unknown		M1
$a = 3$, $d = 5$ (both values)	(c.a.o.)	A1

2008 Summer

4. (*a*) $S_n = a + [a + d] + . . . + [a +(n - 2)d] + [a + (n - 1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + . . . + [a + d] + a$

$2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + . . . + [a + a + (n - 1)d]$

(reverse and add) M1

$2S_n = n[2a + (n - 1)d]$

$S_n = \frac{n}{2}[2a + (n - 1)d]$ (convincing) A1

(*b*) $\frac{10}{2} \times [2a + 9d] = 320$ B1

$2a + 9d = 64$

$[a + 11(12)d] + [a + 15(16)d] = 166$ M1

$[a + 11d] + [a + 15d] = 166$ A1

$2a + 26d = 166$

An attempt to solve the candidate's two equations simultaneously by eliminating one unknown M1

$d = 6, a = 5$ (both values) (c.a.o.) A1

2009 Winter

4. (*a*) $a + 12d = 51$ B1

$a + 8d = k \times (a + d)$ ($k = 5, {}^1/_5$) M1

$a + 8d = 5(a + d)$ A1

$3d = 4a$

An attempt to solve the candidate's equations simultaneously by eliminating one unknown M1

$d = 4, a = 3$ (both values) (c.a.o.) A1

(*b*) $S_{20} = \frac{20}{2} \times (5 + 62)$

(substitution of values in formula for sum of A.P.) M1

$S_{20} = 670$ A1

2009 Summer

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n - 1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + a$

Either:

$2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \ldots + [a + a + (n - 1)d]$ Or:

$2S_n = [a + a + (n - 1)d] + \ldots$ (n times) M1

$2S_n = n[2a + (n - 1)d]$

$S_n = \frac{n}{2}[2a + (n - 1)d]$ (convincing) A1

(*b*) $a + 7d = 46$ B1

$\frac{9}{2} \times [2a + 8d] = 225$ B1

An attempt to solve the candidate's equations simultaneously by eliminating one unknown M1

$a = -3,\ d = 7$ (both values) (c.a.o.) A1

(*c*) $a = 3,\ d = 4$ B1

$S_n = \frac{n}{2}[2 \times 3 + (n - 1)4]$ (f.t. candidate's d) M1

$S_n = n(2n + 1)$ (c.a.o.) A1

2010 Winter

4. (*a*) At least one correct use of the sum formula M1

$\frac{8}{2} \times [2a + 7d] = 124$

$\frac{20}{2} \times [2a + 19d] = 910$ (both correct) A1

An attempt to solve the candidate's two equations simultaneously by eliminating one unknown M1

$d = 5$ (c.a.o.) A1

$a = -2$ (f.t. candidate's value for d) A1

(*b*) $-2 + 5(n - 1) = 183$ (f.t. candidate's values for a and d) M1

$n = 38$ (c.a.o.) A1

2010 Summer

5. (*a*) $S_n = a + [a + d] + \ldots + [a + (n - 1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + a$

Either:

$2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \ldots + [a + a + (n - 1)d]$

Or

$2S_n = [a + a + (n - 1)d] +$ (n times) M1

$2S_n = n[2a + (n - 1)d]$

$S_n = \frac{n}{2}[2a + (n - 1)d]$ (convincing) A1

(*b*) $\frac{n}{2}[2 \times 4 + (n - 1) \times 2] = 460$ M1

Either: Rewriting above equation in a form ready to be solved

$2n^2 + 6n - 920 = 0$ or $n^2 + 3n - 460 = 0$ or $n(n + 3) = 460$

or: $n = 20, n = -23$ A1

$n = 20$ (c.a.o.) A1

(*c*) $a + 4d = 9$ B1

$(a + 5d) + (a + 9d) = 42$ B1

An attempt to solve the candidate's two linear equations

Simultaneously by eliminating one unknown M1

$d = 4$ (c.a.o.) A1

$a = -7$ (f.t. candidate's value for d) A1

2011 Winter

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n-1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n-1)d] + [a + (n-2)d] + \ldots + a$

Either:

$2S_n = [a + a + (n-1)d] + [a + a + (n-1)d] + \ldots + [a + a + (n-1)d]$

Or:

$2S_n = [a + a + (n-1)d] + \ldots$ (n times) M1

$2S_n = n[2a + (n-1)d]$

$S_n = \frac{n}{2}[2a + (n-1)d]$ (convincing) A1

(*b*) $a + 7d = 28$ B1

$\frac{20}{2} \times [2a + 19d] = 710$ B1

An attempt to solve the candidate's equations simultaneously by eliminating one unknown M1

$d = 3$ (c.a.o.) A1

$a = 7$ (f.t. candidate's value for d) A1

(*c*) $S_{15} = \frac{15}{2} \times (-3 + 67)$

(substitution of values in formula for sum of A.P.) M1

$S_{15} = 480$ A1

2011 Summer

4. (*a*) $\frac{15}{2} \times [2a + 14d] = 780$ B1

Either $[a + d] + [a + 3d] + [a + 9d] = 100$

or $[a + 2d] + [a + 4d] + [a + 10d] = 100$ M1

$3a + 13d = 100$ (seen or implied by later work) A1

An attempt to solve candidate's derived linear equations simultaneously by eliminating one unknown M1

$a = 3$, $d = 7$ (both values) (c.a.o.) A1

(*b*) $d = 9$ B1

A correct method for finding $(p + 7)$ th term M1

$(p + 7)$ th term $= 1086$ (c.a.o.) A1

2012 Winter

4. (*a*)

$a + 14d = k \times (a + 4d)$ $\quad (k = 7, {}^1/_7)$ M1

$a + 14d = 7 \times (a + 4d)$ A1

$3a + 7d = 0$

$\frac{11}{2} \times (2a + 10d) = 88$ B1

$a + 5d = 8$

An attempt to solve the candidate's two derived linear equations simultaneously by eliminating one unknown M1

$d = 3$ (c.a.o.) A1

$a = -7$ (f.t. candidate's value for d) A1

(*b*)

$-7 + (n - 1) \times 3 = 143$ (f.t. candidate's values for a and d) M1

$n = 51$ (c.a.o.) A1

2012 Summer

4. (*a*)

$S_n = a + [a + d] + \ldots + [a + (n - 1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + a$

Either:

$2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \ldots + [a + a + (n - 1)d]$

Or:

$2S_n = [a + a + (n - 1)d]$ $\quad n$ times M1

$2S_n = n[2a + (n - 1)d]$

$S_n = \frac{n}{2}[2a + (n - 1)d]$ (convincing) A1

(*b*)

$a + 2d + a + 3d + a + 9d = 79$ B1

$a + 5d + a + 6d = 61$ B1

An attempt to solve the candidate's linear equations simultaneously by eliminating one unknown M1

$a = 3, d = 5$ (both values) (c.a.o.) A1

(*c*)

$a = 15, d = -2$ B1

$S_n = \frac{n}{2}[2 \times 15 + (n - 1)(-2)]$ (f.t. candidate's d) M1

$S_n = n(16 - n)$ (c.a.o.) A1

2013 Winter

4. (*a*) (i) nth term $= 1 + 4(n - 1) = 1 + 4n - 4 = 4n - 3$ (convincing) B1

(ii) $S_n = 1 + 5 + \ldots + (4n - 7) + (4n - 3)$

$S_n = (4n - 3) + (4n - 7) + \ldots + 5 + 1$

Reversing and adding M1

Either:

$2S_n = (4n - 2) + (4n - 2) + \ldots + (4n - 2) + (4n - 2)$

Or:

$2S_n = (4n - 2) + \ldots$ (n times) A1

$2S_n = n(4n - 2)$

$S_n = n(2n - 1)$ (convincing) A1

(*b*) $\frac{10}{2} \times [2a + 9d] = 55$ B1

Either: $(a + 3d) + (a + 6d) + (a + 8d) = 27$

Or: $(a + 4d) + (a + 7d) + (a + 9d) = 27$ M1

$3a + 17d = 27$ (seen or implied by later work) A1

An attempt to solve candidate's derived linear equations simultaneously by eliminating one unknown M1

$a = -8$, $d = 3$ (both values) (c.a.o.) A1

2013 Summer

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n - 1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + a$

Either:

$2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \ldots + [a + a + (n - 1)d]$

(at least three terms, including those derived from the first pair and the last pair plus one other pair of terms)

Or:

$2S_n = [a + a + (n - 1)d] + \ldots$ (n times) M1

$2S_n = n[2a + (n - 1)d]$

$S_n = \frac{n}{2}[2a + (n - 1)d]$ (convincing) A1

2014 Winter

3. (*a*) $a + 2d + a + 7d = 0$ B1

$a + 4d + a + 6d + a + 9d = 22$ B1

An attempt to solve the candidate's linear equations simultaneously by eliminating one unknown M1

$a = -18,\ d = 4$ (both values) (c.a.o.) A1

(*b*) $S_n = \frac{n}{2}[2 \times 9 + (n - 1) \times 2]$ B1

$S_{2n} = \frac{2n}{2}[2 \times 9 + (2n - 1) \times 2]$ B1

$\frac{2n}{2}[2 \times 9 + (2n - 1) \times 2] = k \times \frac{n}{2}[2 \times 9 + (n - 1) \times 2]$ ($k = \mathbf{3}, {}^1/_3$)

(f.t. candidate's quadratic expressions for S_{2n}, S_n provided at least one of the first two B marks is awarded) M1

An attempt to solve this equation including dividing both sides by n to reach a linear equation in n. m1

$n = 8$ (c.a.o.) A1

2014 Summer

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n - 1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + a$

In order to make further progress, the two expressions for S_n must contain at least three pairs of terms, including the first pair, the last pair and one other pair of terms

Either:

$2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \ldots + [a + a + (n - 1)d]$

Or:

$2S_n = [a + a + (n - 1)d]$ n times M1

$2S_n = n[2a + (n - 1)d]$

$S_n = \frac{n}{2}[2a + (n - 1)d]$ (convincing) A1

(*b*) $\frac{n}{2}[2 \times 3 + (n - 1) \times 2] = 360$ M1

Rewriting above equation in a form ready to be solved

$2n^2 + 4n - 720 = 0$ or $n^2 + 2n - 360 = 0$ or $n(n + 2) = 360$ A1

$n = 18$ (c.a.o.) A1

(*c*) $a + 9d = 7 \times (a + 2d)$ B1

$a + 7d + a + 8d = 80$ B1

An attempt to solve the candidate's linear equations simultaneously by eliminating one unknown M1

$a = -5, d = 6$ (both values) (c.a.o.) A1

2015

4. (*a*) (i) nth term $= 4 + 6(n-1) = 4 + 6n - 6 = 6n - 2$ (convincing) B1

(ii) $S_n = 4 + 10 + \ldots + (6n-8) + (6n-2)$

$S_n = (6n-2) + (6n-8) + \ldots + 10 + 4$

Reversing and adding M1

Either:

$2S_n = (6n+2) + (6n+2) + \ldots + (6n+2) + (6n+2)$

Or:

$2S_n = (6n+2) + \ldots$ (n times) A1

$2S_n = n(6n+2)$

$S_n = n(3n+1)$ (convincing) A1

(*b*) (i) $a + 9d = 4 \times (a + 4d)$ B1

$3a + 7d = 0$

$\frac{15}{2} \times (2a + 14d) = 210$ B1

$a + 7d = 14$

An attempt to solve the candidate's two derived linear equations simultaneously by eliminating one unknown M1

$d = 3, a = -7$ (c.a.o.) A1

(ii) $-7 + (k-1) \times 3 = 200$

(f.t. candidate's derived values for a and d) M1

$k = 70$ (c.a.o.) A1

2016

4. (*a*) This is an A.P. with $a = 6, d = 2$ (s.i.) M1

(i) 20th term $= 6 + 2 \times 19$

(f.t. candidate's values for a and d) M1

20th term $= 44$ (c.a.o.) A1

(ii) $\frac{n}{2}[2 \times 6 + (n-1) \times 2] = 750$

(f.t. candidate's values for a and d) M1

Rewriting above equation in a form ready to be solved

$2n^2 + 10n - 1500 = 0$ or $n^2 + 5n - 750 = 0$ or $n(n+5) = 750$ or

$n^2 + 5n = 750$ (f.t. candidate's values for a and d) A1

$n = 25$ (c.a.o.) A1

(*b*) (i) $t_{11} + t_{14} = 50$ B1

(ii) $S_{24} = \frac{24}{2} \times 50$ M1

$S_{24} = 600$ A1

2017

4. (*a*) $S_n = a + [a + d] + \ldots + [a + (n - 1)d]$

(at least 3 terms, one at each end) B1

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots + a$

In order to make further progress, the two expressions for S_n must contain at least three pairs of terms, including the first pair, the last pair and one other pair of terms

Either:

$2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \ldots + [a + a + (n - 1)d]$

Or:

$2S_n = [a + a + (n - 1)d]$ n times M1

$2S_n = n[2a + (n - 1)d]$

$S_n = \frac{n}{2}[2a + (n - 1)d]$ (convincing) A1

(*b*) $\frac{8}{2} \times (2a + 7d) = 156$ B1

$2a + 7d = 39$

$\frac{16}{2} \times (2a + 15d) = 760$ B1

$2a + 15d = 95$

An attempt to solve the candidate's two derived linear equations simultaneously by eliminating one unknown M1

$d = 7, a = -5$ (c.a.o.) A1

(*c*) $d = 9$ B1

A correct method for finding $(p + 8)$ th term M1

$(p + 8)$ th term $= 2129$ (c.a.o.) A1