Specimen

4.	(*a*)	$S_n = a + ar + \ldots + ar^{n-2} + ar^{n-1}$	B1
		$rS_n = \quad ar + ar^2 + \ldots + ar^{n-2} + ar^{n-1} + ar^n$	M1
		$\therefore S_n - rS_n = a - ar^n$	
		$\therefore S_n = \dfrac{a(1-r^n)}{1-r}$	A1(convincing)
		Sum to infinity = $\dfrac{a}{1-r}$	B1
	(*b*)	$\dfrac{a}{1-r} = 4a$	M1 ($\dfrac{a}{1-r} = ka$, $k = \dfrac{1}{4}$ or 4)
			A1 (correct)
		$\therefore 1 - r = \dfrac{1}{4}$	M1 (eliminate a and attempt to solve)
		giving $r = \dfrac{3}{4}$	A1

2005 Winter

(8) a) Term cyntaf a
Ail derm ar
Trydydd term ar^2
ac yn y blaen.

Swm yr n term cyntaf mewn cyfres geometrig yw
$S_n = a + ar + ar^2 + \ldots + ar^{n-1}$
Yn lluosi bob ochr ag r
$rS_n = ar + ar^2 + ar^3 + \ldots + ar^n$

Yn tynnu'r ail hafaliad o'r hafaliad cyntaf:
$S_n - rS_n = a - ar^n$
(mae'r termau yn y canol yn canslo mewn parau)

Felly $S_n(1-r) = a - ar^n$
$S_n = \dfrac{a(1-r^n)}{1-r}$.

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2005 Summer

4.	(a)	$a + ar = 6.4, \quad \dfrac{a}{1-r} = 10$		B1, B1
		Eliminate a	$10(1-r)(1+r) = 6.4$	M1 (reasonable attempt to eliminate a)
				A1 (C.A.O.) (any correct expression in r or a)
			$1 - r^2 = 0.64$	
			$r = 0.6$	A1 (F.T. one slip if one B earned)
	(b)	$a = 10 \times (1 - 0.6) = 4$		A1 (F.T. value of r if one B earned)
		$S_{11} = \dfrac{4}{0.4}\left(1-(0.6)^{11}\right) = 9.964$		M1 (use of correct formula with derived a, r)
				A1 (C.A.O.)
				[8]

2006 Winter

4.	*(a)*	nth term $= ar^{n-1}$	B1
		Let $S_n = a + ar + ar^2 + \ldots + ar^{n-2} + ar^{n-1}$ (1)	B1 (at least 3 terms, one at each end)
		$rS_n = ar + ar^2 + \ldots + ar^{n-2} + ar^{n-1} + ar^n$ (2)	M1 (multiplication by r and subtract)
	(1) – (2)		
		$S_n - rS_n = a - ar^n$	
		$S_n(1 - r) = a(1 - r^n)$	
		$S_n = \dfrac{a(1-r^n)}{1-r}$	A1 (convincing)

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2006 Summer

5.	*(a)*	$ar = 9\,ar^3$	M1 ($ar = kar^3$, $k = 9, \frac{1}{9}$)
			A1 (correct)
		$1 = 9r^2$	A1 (F.T. value of k)
		$r = \pm\,\frac{1}{3}$	A1 (F.T. value of k, $r = \pm\,3$)

	(b)	$\frac{a}{1-\frac{1}{3}} = 12$	M1 (use of correct formula)
		$a = 8$	A1 (F.T. derived r)
		Third term $= 8 \times \left(\frac{1}{3}\right)^2 = \frac{8}{9}$	(F.T. r) A1

7

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2007 Winter

3. (*a*) n'th term $= ar^{n-1}$ B1

$S_n = a + ar + \ldots + ar^{n-2} + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = ar + ar^2 + \ldots ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply by r and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \dfrac{a(1 - r^n)}{1 - r}$ (convincing) A1

$S_\infty = \dfrac{a}{1 - r}$ B1

(*b*) (i) $a + ar = k(ar + ar^2)$ ($k = 2, \frac{1}{2}$) M1

$1 + r = 2r + 2r^2$ A1

An attempt to collect terms, form and solve quadratic equation M1

$2r^2 + r - 1 = 0 \Rightarrow (2r - 1)(r + 1) = 0 \Rightarrow r = \frac{1}{2},$ A1

(ii) $r = \frac{1}{2} \Rightarrow \dfrac{a}{1 - \frac{1}{2}} = 12$

(f.t. candidate's value for r provided $0 < r < 1$) B1

$a = 6$ (f.t. candidate's value for r provided $0 < r < 1$) B1

$S_8 = \dfrac{6}{\frac{1}{2}}\{1 - (\frac{1}{2})^8\}$ M1

$S_8 \approx 11{\cdot}95$ (f.t. candidate's derived values of a, r) A1

2007 Summer

5. (*a*) $S_n = a + ar + \ldots + ar^{n-2} + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = ar + ar^2 + \ldots ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by r and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \dfrac{a(1 - r^n)}{1 - r}$ (convincing) A1

$S_\infty = \dfrac{a}{1 - r}$ B1

(*b*) (i) $\dfrac{a}{1 - r} = 10$ B1

$\dfrac{a}{1 - 2r} = 15$ B1

An attempt to eliminate a M1

$r = 0{\cdot}25$ (c.a.o.) A1

(ii) $a = 7{\cdot}5$ B1

$S_4 = \dfrac{7{\cdot}5[1 - 0{\cdot}25^4]}{1 - 0{\cdot}25}$

(award even if sum calculated for 2nd series) M1

$S_4 \approx 9{\cdot}96$ (f.t. candidate's derived values of a, r) A1

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2008 Winter

4.	(*a*)	$ar^7 = 5$ and $ar^4 = 135$		M1
		$r^3 = \dfrac{5}{135}$		m1
		$r = \dfrac{1}{3}$		A1
		$a \times \dfrac{1}{3^4} = 135$		M1
		$a = 10935$	(c.a.o.)	A1
	(*b*)	$S_\infty = \dfrac{10935}{1 - {}^1/_3}$	(use of formula for sum to infinity)	M1
		$S_\infty = 16402{\cdot}5$	(f.t. candidate's derived value for *a*)	A1

2008 Summer

5.	$a + ar = 7{\cdot}2$		B1
	$\dfrac{a}{1 - r} = 20$		B1
	A valid attempt to eliminate a		M1
	$20(1 - r) + 20(1 - r)r = 7{\cdot}2$	(a correct quadratic in r)	A1
	$r = 0{\cdot}8$ $\quad (r = -0{\cdot}8)$	(c.a.o.)	A1
	$r = 0{\cdot}8$ and $a = 4$	(f.t. candidate's positive value for r)	A1

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2009 Winter

5. (*a*) $S_n = a + ar + \ldots + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by r and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \dfrac{a(1 - r^n)}{1 - r}$ (convincing) A1

(*b*) $r = 0{\cdot}9$ B1

$S_{18} = \dfrac{10(1 - 0{\cdot}9^{18})}{1 - 0{\cdot}9}$ (f.t. candidate's numerical value for r) M1

$S_{18} \approx 84{\cdot}990 = 85$ (c.a.o.) A1

(*c*) (i) $ar = -4$ B1

$\dfrac{a}{1 - r} = 9$ B1

An attempt to solve these equations simultaneously by eliminating a M1

$9r^2 - 9r - 4 = 0$ (convincing) A1

(ii) $r = -{}^1/_3$ (c.a.o.) B1

$|\,r\,| < 1$ E1

2009 Summer

5. (*a*) $r = \dfrac{108}{36} = 3$ (c.a.o.) B1

$t_7 = \dfrac{36}{3^2}$ (f.t. candidate's value for r) M1

$t_7 = 4$ (c.a.o.) A1

(*b*) (i) $ar = 9$ B1

$\dfrac{a}{1 - r} = 48$ B1

An attempt to solve these equations simultaneously by eliminating a M1

$16r^2 - 16r + 3 = 0$ (convincing) A1

(ii) $r = {}^1/_4, {}^3/_4$ B1

$a = 36, 12$ (c.a.o.) B1

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2010 Winter

5.	(*a*)	$S_n = a + ar + \ldots + ar^{n-1}$	(at least 3 terms, one at each end)	B1
		$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$		
		$S_n - rS_n = a - ar^n$	(multiply first line by r and subtract)	M1
		$(1 - r)S_n = a(1 - r^n)$		
		$S_n = \dfrac{a(1 - r^n)}{1 - r}$	(convincing)	A1
	(*b*)	**Either:** $\dfrac{a(1 - r^4)}{1 - r} = 73{\cdot}8$		
		Or: $a + ar + ar^2 + ar^3 = 73{\cdot}8$		B1
		$\dfrac{a}{1 - r} = 125$		B1
		An attempt to solve these equations simultaneously by eliminating one of the variables		M1
		$r^4 = 0{\cdot}4096$		A1
		$r = 0{\cdot}8$	(c.a.o.)	A1
		$a = 25$	(f.t. candidate's value for r)	A1

2010 Summer

6.	(*a*)	$r = -0{\cdot}6$		B1
		$S_\infty = \dfrac{40}{1 - (-0{\cdot}6)}$		M1
		$S_\infty = 25$	(c.a.o.)	A1
	(*b*)	(i) $ar^3 = 8$		B1
		$ar^2 + ar^3 + ar^4 = 28$		B1
		An attempt to solve these equations simultaneously by eliminating a		M1
		$\dfrac{r^3}{r^2 + r^3 + r^4} = \dfrac{8}{28} \Rightarrow 2r^2 - 5r + 2 = 0$	(convincing)	A1
		(ii) $r = 0{\cdot}5$ ($r = 2$ discarded, c.a.o.)		B1
		$a = 64$ (f.t. candidate's value for r, provided $\lvert r \rvert < 1$)		B1

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2011 Winter

5. (*a*) (i) $ar = 6$ and $ar^4 = 384$ — B1

$r^3 = \frac{384}{6}$ (o.e.) — M1

$r = 4$ (c.a.o.) — A1

(ii) $a \times 4 = 6 \Rightarrow a = 1{\cdot}5$ — B1

$S_8 = \frac{1{\cdot}5(4^8 - 1)}{4 - 1}$ (correct use of formula for S_8, f.t. candidate's derived values for r and a) — M1

$S_8 = 32767{\cdot}5$ (f.t. candidate's derived values for r and a) — A1

(*b*) (i) $5 \times 1{\cdot}1^{n-1} = 170$ — M1

$1{\cdot}1^{n-1} = 34$ — A1

$(n - 1)\log 1{\cdot}1 = \log 34$ (f.t. only $5{\cdot}5^{n-1} = 170$ or $1{\cdot}1^n = 34$) — M1

$n = 38$ (c.a.o.) — A1

(ii) $|r|$ must be < 1 for sum to infinity to exist — E1

2011 Summer

5. (*a*) $S_n = a + ar + \ldots + ar^{n-1}$ (at least 3 terms, one at each end) — B1

$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by r and subtract) — M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \frac{a(1 - r^n)}{1 - r}$ (convincing) — A1

(*b*) (i) $\frac{a}{1 - r} = ka,$ ($k = 4$ or ¼) — M1

$r = 0{\cdot}75$ (c.a.o.) — A1

(ii) $a + 0{\cdot}75a = 35$ (f.t. candidate's derived value for r, provided $r \neq 1$) — M1

$a = 20$ (f.t. candidate's derived value for r, provided $r \neq 1$) — A1

$S_9 = \frac{20(1 - 0{\cdot}75^9)}{1 - 0{\cdot}75}$ (f.t. candidate's derived values for r and a, provided $r \neq 1$) — M1

$S_9 = 73{\cdot}99 = 74$ (c.a.o.) — A1

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2012 Winter

5. (*a*) $S_n = a + ar + \ldots + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by r and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \dfrac{a(1 - r^n)}{1 - r}$ (convincing) A1

(*b*) $a + ar = 25{\cdot}2$ or $\dfrac{a(1 - r^2)}{(1 - r)} = 25{\cdot}2$ B1

$\dfrac{a}{1 - r} = 30$ B1

An attempt to solve the candidate's derived equations simultaneously by eliminating a M1

$30(1 - r) + 30(1 - r)r = 25{\cdot}2$ (a correct quadratic in r) A1

$r = 0{\cdot}4$ (c.a.o.) A1

$a = 18$ (f.t. candidate's value for r provided $r > 0$) A1

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2012 Summer

5. (*a*) $a + ar = 72$ B1

$a + ar^2 = 120$ B1

An attempt to solve candidate's equations simultaneously by correctly eliminating a M1

$3r^2 - 5r - 2 = 0$ (convincing) A1

(*b*) An attempt to solve quadratic equation in r, either by using the quadratic formula or by getting the expression into the form $(ar + b)(cr + d)$, with $a \times c = 3$ and $b \times d = -2$ M1

$(3r + 1)(r - 2) = 0 \Rightarrow r = -{}^{1}/_{3}$ A1

$a \times (1 - {}^{1}/_{3}) = 72 \Rightarrow a = 108$ (f.t. candidate's derived value for r) B1

$S_\infty = \dfrac{108}{1 - (-{}^{1}/_{3})}$ (correct use of formula for S_∞, f.t. candidate's derived values for r and a) M1

$S_\infty = 81$ (c.a.o.) A1

2013 Winter

5. (*a*) $r = 1{\cdot}5$ B1

A correct method for finding $(p + 4)$ th term M1

$(p + 4)$ th term $= 81$ (c.a.o.) A1

(*b*) Either: $\dfrac{a(1 - r^3)}{1 - r} = 22{\cdot}8$

Or: $a + ar + ar^2 = 22{\cdot}8$ B1

$\dfrac{a}{1 - r} = 18{\cdot}75$ B1

An attempt to solve these equations simultaneously by eliminating a M1

$r^3 = -0{\cdot}216$ A1

$r = -0{\cdot}6$ (c.a.o.) A1

$a = 30$ (f.t. candidate's derived value for r) A1

2013 Summer

5. (*a*) $r = 0{\cdot}8$ B1

$S_{18} = \dfrac{100(1 - 0{\cdot}8^{18})}{1 - 0{\cdot}8}$ M1

$S_{18} \approx 490{\cdot}992 = 491$ (c.a.o.) A1

(*b*) (i) $ar = -20$ B1

$\dfrac{a}{1 - r} = 64$ B1

An attempt to solve these equations simultaneously by eliminating a M1

$16r^2 - 16r - 5 = 0$ (convincing) A1

(ii) $r = -\frac{1}{4}$ (c.a.o.) B1

$|r| < 1$ E1

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2014 Winter

5. (*a*) $ar + ar^2 = -216$ B1

$ar^4 + ar^5 = 8$ B1

A correct method for solving the candidate's equations simultaneously

e,g multiplying the first equation by r^3 and subtracting

or eliminating a and $(1 + r)$ M1

$-216r^3 = 8$ (o.e.) A1

$r = -{}^1/_3$ (convincing) A1

(*b*) $a \times (-{}^1/_3) \times (1 - {}^1/_3) = -216 \Rightarrow a = 972$ B1

$S_\infty = \dfrac{972}{1 - (-{}^1/_3)}$ (correct use of formula for S_∞, f.t. candidate's derived value for a) M1

$S_\infty = 729$ (f.t. candidate's derived value for a) A1

2014 Summer

5. (*a*) $ar + ar^2 = -216$ B1

$ar^4 + ar^5 = 8$ B1

A correct method for solving the candidate's equations simultaneously

e,g multiplying the first equation by r^3 and subtracting

or eliminating a and $(1 + r)$ M1

$-216r^3 = 8$ (o.e.) A1

$r = -{}^1/_3$ (convincing) A1

(*b*) $a \times (-{}^1/_3) \times (1 - {}^1/_3) = -216 \Rightarrow a = 972$ B1

$S_\infty = \dfrac{972}{1 - (-{}^1/_3)}$ (correct use of formula for S_∞, f.t. candidate's derived value for a) M1

$S_\infty = 729$ (f.t. candidate's derived value for a) A1

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5. (*a*) $r = \frac{2304}{576} = 4$ (c.a.o.) B1

$t_5 = \frac{576}{4^3}$ (f.t. candidate's value for r) M1

$t_5 = 9$ (c.a.o.) A1

(*b*) (i) $ar^2 = 24$ B1

$ar + ar^2 + ar^3 = -56$ B1

An attempt to solve the candidate's equations simultaneously by eliminating a M1

$\frac{r^2}{r + r^2 + r^3} = -\frac{24}{56} \Rightarrow 3r^2 + 10r + 3 = 0$ (convincing) A1

(ii) $r = -\frac{1}{3}$ ($r = -3$ discarded, c.a.o.) B1

$a = 216$

(f.t. candidate's derived value for r, provided $|r| < 1$) B1

$S_\infty = \frac{216}{1 - (-{}^1/_3)}$ (use of formula for sum to infinity)

(f.t. candidate's derived values for r and a) M1

$S_\infty = 162$ (f.t. candidate's derived values for r and a) A1

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2016

5. (*a*) $a = 2, \ b = -12$ B1 B1

(*b*) $x^2 + 4x - 8 = 2x + 7$ M1

An attempt to collect terms, form and solve the quadratic equation in x either by correct use of the quadratic formula or by writing the equation in the form $(x + n)(x + m) = 0$, where $n \times m =$ candidate's constant m1

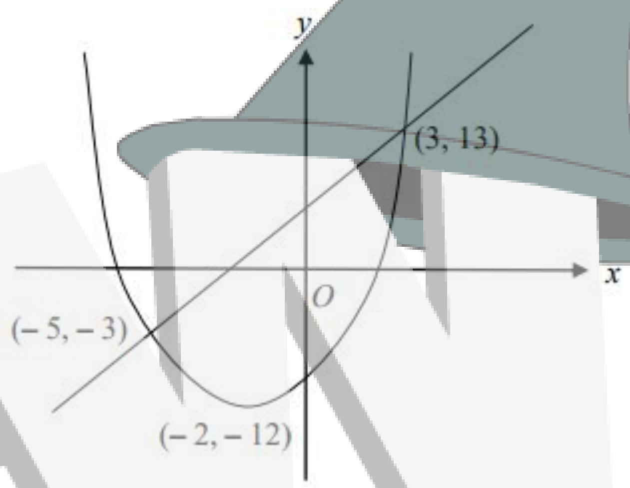
$x^2 + 2x - 15 = 0 \Rightarrow (x - 3)(x + 5) = 0 \Rightarrow x = 3, x = -5$

(both values, c.a.o.) A1

When $x = 3, y = 13$, when $x = -5, y = -3$

(both values, f.t. one slip) A1

(*c*)



A positive quadratic graph M1

Minimum point $(-2, -12)$ marked

(f.t. candidate's values for a, b) A1

A straight line with positive gradient and positive y-intercept B1

Both points of intersection $(-5, -3)$, $(3, 13)$ marked

(f.t candidate's solutions to part(*b*)) B1

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5. (*a*) $a = 100,\ r = 1{\cdot}2$

Value of donation in 12^{th} year $= 100 \times 1{\cdot}2^{11}$ M1

Value of donation in 12^{th} year = £743 A1

(*b*) $100 \times \dfrac{(1 - 1{\cdot}2^n)}{1 - 1{\cdot}2} = 15474$ M1

$1 - 1{\cdot}2^n = 154{\cdot}74 \times (-\,0{\cdot}2)$ m1

$1{\cdot}2^n = 31{\cdot}948$ A1

$n = \dfrac{\log 31{\cdot}948}{\log 1{\cdot}2}$ m1

$n = 19$ cao A1

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