C3 Differentiation Questions

Specimen

6. Differentiate the following with respect to x, simplifying your answers as much as possible.

(a) $e^{2x} \sin x$

(b) $\dfrac{2x^2 - 4}{x^2 + 3}$

(c) $\tan(4x^2 + 3)$ [4], [3], [2]

8. *(a)* Given that $y = \tan^{-1}x$, show that

$$\frac{dy}{dx} = \frac{1}{x^2 + 1}.$$ [3]

(b) Differentiate $\ln(x^2 + 1)$ with respect to x. [2]

(c) Use the results derived in *(a)* and *(b)* to find

$$\int \frac{3 + x}{1 + x^2} dx.$$ [4]

2005 Summer

6. *(a)* Differentiate each of the following with respect to x and simplify your answers.

(i) e^{2x-5} (ii) $x^2 \ln x$ (iii) $(3x^2 + 2)^4$ [8]

(b) By first writing $\tan x = \dfrac{\sin x}{\cos x}$, show that $\dfrac{d}{dx}(\tan x) = \sec^2 x$. [3]

(c) By first writing $y = \tan^{-1} x$ as $x = \tan y$, show that $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1 + x^2}$. [3]

2006 Winter

5. Differentiate each of the following with respect to x, simplifying your answer where possible.

(a) $e^{3x}\cos x$ [3]

(b) $\dfrac{2x^2+1}{3x^2+2}$ [3]

(c) $\tan(5x^2+3)$ [2]

(d) $\ln(2x)$ [2]

(e) $\sin^{-1}(3x)$. [2]

2006 Summer

5. *(a)* Differentiate each of the following with respect to x,

(i) $\tan^{-1}4x$ (ii) $\ln(1+x^2)$ (iii) x^2e^{3x} [7]

(b) By first writing $\cot x = \dfrac{\cos x}{\sin x}$, show that $\dfrac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$. [3]

2007 Winter

4. Differentiate each of the following with respect to x and simplify your answers where possible.

(a) $(1+2x)^{15}$ *(b)* $\ln(1+x^2)$ *(c)* $\dfrac{2+\cos x}{1+\sin x}$

(d) $\tan^{-1}(3x)$ *(e)* $x^2\tan x$ [2], [2], [3], [2], [2]

2007 Summer

6. *(a)* Differentiate each of the following with respect to x and simplify your answers, wherever possible.

(i) $x^2\sin x$ (ii) $\ln(x^2+3)$ (iii) e^{9-2x} (iv) $\dfrac{4}{(3x+7)^2}$

(v) $\sin^{-1}3x$ [10]

(b) Given $y=\dfrac{1+\tan x}{1-\tan x}$ $(\tan x \neq 1)$, show that $\dfrac{dy}{dx}$ is always positive. [4]

2008 Winter

5. Differentiate each of the following with respect to x, simplifying your answers wherever possible.

(a) $\dfrac{\ln x}{x^2}$ *(b)* $\cos^{-1} 5x$ *(c)* $\sqrt{1+6x^4}$

(d) $x^3\tan 2x$ [3],[2],[2],[3]

2008 Summer

8. Differentiate *(a)* $\cot 2x$, *(b)* $x^2 \ln x$, *(c)* $\dfrac{x^2+1}{x^2-2}$,

simplifying your answers wherever possible. [2], [2], [3]

2009 Winter

5. *(a)* Differentiate **each** of the following with respect to x and simplify your answers, wherever possible.

(i) $\ln(\sin x)$ (ii) $\sin^{-1}(4x)$ (iii) $\dfrac{3x^2+2}{x^2+5}$ [8]

(b) By first writing $y = \tan^{-1}x$ as $x = \tan y$, find $\dfrac{dy}{dx}$ in terms of x. [4]

2009 Summer

5. Differentiate each of the following with respect to x, simplifying your answers where possible.

(a) $\ln(3+2x^2)$ *(b)* $x^2\tan^{-1}x$ *(c)* $(5+7x^2)^{10}$ [2], [2], [3]

2010 Winter

5. Differentiate **each** of the following with respect to x, simplifying your answer wherever possible.

(a) $\tan^{-1} 3x$ *(b)* $\ln(2x^2 - 3x + 4)$ [2], [2]

(c) $e^{2x} \sin x$ *(d)* $\dfrac{1 - \cos x}{1 + \cos x}$ [3], [3]

2010 Summer

5. *(a)* Differentiate **each** of the following with respect to x, simplifying your answer wherever possible.

(i) $(7 + 2x)^{13}$ (ii) $\sin^{-1} 5x$ (iii) $x^3 e^{4x}$ [7]

(b) By first writing $\tan x = \dfrac{\sin x}{\cos x}$, show that

$$\frac{d}{dx}(\tan x) = \sec^2 x.$$ [3]

2011 Winter

5. *(a)* Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(i) $\sqrt{2 + 5x^3}$ (ii) $x^2 \sin 3x$ (iii) $\dfrac{e^{2x}}{x^4}$ [8]

(b) By first writing $y = \tan^{-1} x$ as $x = \tan y$, find $\dfrac{dy}{dx}$ in terms of x. [4]

2011 Summer

5. *(a)* Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(i) $\sqrt{2 + 5x^3}$ (ii) $x^2 \sin 3x$ (iii) $\dfrac{e^{2x}}{x^4}$ [8]

(b) By first writing $y = \tan^{-1} x$ as $x = \tan y$, find $\dfrac{dy}{dx}$ in terms of x. [4]

2012 Winter

4. Given that $x^2y^2 + x^4 + 6 = 2y^3 + 2x$, find the value of $\frac{dy}{dx}$ at the point (2, 3). [4]

5. Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(a) $\tan^{-1}4x$ *(b)* e^{x^3} [2], [2]

(c) $x^5\ln x$ *(d)* $\frac{3-2x^2}{5-4x^2}$ [3], [3]

2012 Summer

3. *(a)* The curve C is defined by

$$x^3 - 4x^2y = 2y^3 - 3x - 2.$$

Find the value of $\frac{dy}{dx}$ at the point (3, 1). [4]

5. Differentiate each of the following with respect to x.

(a) $\ln(7 + 2x - 3x^2)$ *(b)* $e^{\tan x}$ *(c)* $5x^2\sin^{-1}x$ [2], [2], [3]

2013 Winter

3. *(a)* Given that

$$x^3 + 5x^4y - 2y^3 + 7 = 0,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

5. *(a)* Differentiate each of the following with respect to x.

(i) $\sqrt{5x^2 - 3x}$ (ii) $\sin^{-1}7x$ (iii) $e^{3x}\ln x$ [7]

(b) By first writing $\cot x = \frac{\cos x}{\sin x}$, show that $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$. [3]

2013 Summer

3. The curve C is defined by

$$x^3y^2 = 128.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y. [3]

The point P lies on C and has coordinates (a, b).

(b) Given that the value of $\frac{dy}{dx}$ at the point P is 3,

(i) show that $b = -2a$,

(ii) find the value of a and the value of b. [4]

5. Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(a) $(7 - 9x^2)^5$ *(b)* $\tan^{-1} 6x$ [2], [2]

(c) $e^{4x}\tan 2x$ *(d)* $\frac{3 + \sin x}{2 + \cos x}$ [3], [3]

2014 Winter

3. The curve C is defined by

$$x^3 - 2x^2y + 3y^2 = 3.$$

Find the value of $\frac{dy}{dx}$ at the point $(-2, -1)$. [4]

6. Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(a) $(5x^3 - x)^{10}$ *(b)* $\sin^{-1}(x^3)$ [2], [2]

(c) $x^4\ln(2x)$ *(d)* $\frac{e^{4x}}{(2x + 3)^6}$ [3], [4]

2014 Summer

3. The curve *C* is defined by

$$y^4 - 2x^2 + 8xy^2 + 9 = 0.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{x - 2y^2}{y^3 + 4xy}$. [4]

(b) Show that there is no point on *C* at which $\dfrac{dy}{dx} = 0$. [4]

6. *(a)* Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(i) $\dfrac{1}{\sqrt[4]{9 - 4x^5}}$ (ii) $\dfrac{3 + 2x^3}{7 - x^3}$ [5]

(b) (i) Sketch the graph of $y = \sin^{-1}x$ for values of x satisfying $-1 \leqslant x \leqslant 1$.

(ii) By first rewriting $y = \sin^{-1}x$ as $x = \sin y$, find an expression for $\dfrac{dy}{dx}$ in terms of x. You should justify any choice of sign that you make. [6]

2015

3. *(a)* The curve C_1 is defined by

$$x^3 + 2x\cos y + y^2 = 1 + \frac{\pi^2}{4}.$$

Find the value of $\dfrac{dy}{dx}$ at the point $(1, \frac{\pi}{2})$. [4]

(b) The curve C_2 is such that

$$\frac{dy}{dx} = x^2 y.$$

Find an expression for $\dfrac{d^2y}{dx^2}$ in terms of x and y. Simplify your answer. [3]

6. *(a)* Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(i) $\ln(4x^2 - 3x - 5)$

(ii) $e^{\sqrt{x}}$

(iii) $\dfrac{a + b\sin x}{a - b\sin x}$, where a, b are constants. [7]

(b) By first writing $\cot x = (\tan x)^{-1}$ and assuming the derivative of $\tan x$, find an expression for $\dfrac{d}{dx}(\cot x)$. Simplify your answer. [3]

2016

3. The curve C is defined by

$$x^2 + 3xy + 2y^3 - 2x = 21.$$

The point P has coordinates (– 5, 2) and lies on C.

Find the value of $\frac{dy}{dx}$ at P. [4]

6. Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(a) $\ln(\cos x)$ [3]

(b) $\tan^{-1}(\frac{x}{3})$ [3]

(c) $e^{6x}(3x - 2)^4$ [4]

2017

3. *(a)* Given that

$$x^4 - 3x^2y + 2y^3 - 4x = 7,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

5. *(a)* Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(i) $\sqrt{3x^2 + 5x}$ (ii) $\sin^{-1} 3x$ [4]

(b) By first writing $y = \cot^{-1} x$ as $x = \cot y$ and then assuming the derivative of cot y, find $\frac{dy}{dx}$ in terms of x. [4]