C3 Iterative Loops Questions

Specimen

1. Show that the equation

$$x^3 + 10x - 4 = 0$$

has a root α between 0 and 1.

The iterative formula

$$x_{n+1} = \frac{4 - x_n^3}{10}$$

with $x_0 = 0{\cdot}3$ may be used to find α.

Calculate and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and show that it is the value of α correct to five decimal places. [7]

2005 Summer

2. *(a)* Sketch the graphs of $y = x^4$ and $y = 1 - 3x$. Deduce the number of real roots of the equation

$$x^4 + 3x - 1 = 0.$$ [3]

(b) Show that the equation

$$x^4 + 3x - 1 = 0$$

has a root α between 0 and 1.

The recurrence relation

$$x_{n+1} = \frac{1 - x_n^4}{3}$$

with $x_0 = 0{\cdot}3$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this value is the value of α correct to five decimal places. [7]

2006 Winter

4. *(a)* Show that $2\tan^{-1}x - 6\ln(1+x^2) - 4x^2$ has a stationary value when x satisfies

$$4x^3 + 10x - 1 = 0.$$ [5]

(b) Show that the equation

$$4x^3 + 10x - 1 = 0$$

has a root α between 0 and 1.

The recurrence relation

$$x_{n+1} = \frac{1 - 4x_n^3}{10}$$

with $x_0 = 0{\cdot}1$ may be used to find α. Calculate and record the values of x_1, x_2, x_3. Write down the value of x_3 correct to six decimal places and show that it is the value of α correct to six decimal places. [7]

2006 Summer

(b) Show that the equation

$$e^{2a} - a - 10 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$a_{n+1} = \frac{1}{2}\ln(a_n + 10)$$

with $a_0 = 1{\cdot}2$ can be used to find α. Find and record the values of a_1, a_2, a_3, a_4.

Write down the value of a_4 correct to five decimal places and prove that this value is the value of α correct to five decimal places. [7]

2007 Winter

3. Show that the equation

$$\cos x + 2x - 2 = 0$$

has a root α between 0 and $\frac{\pi}{2}$.

The recurrence relation

$$x_{n+1} = 1 - \frac{1}{2}\cos x_n$$

with $x_0 = 0{\cdot}5$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to three decimal places and prove that this is the value of α correct to three decimal places. [7]

2007 Summer

(b) Show that the equation

$$t^3 + 4t - 2 = 0$$

has a root α between 0 and 1.

The recurrence relation

$$t_{n+1} = \frac{2 - t_n^3}{4}$$

with $t_0 = 0{\cdot}5$ can be used to find α. Find and record the values of t_1, t_2, t_3, t_4. Write down the value of t_4 correct to four decimal places and prove that this value is the value of α correct to four decimal places. [7]

2008 Winter

4. Show that the equation

$$2\ln(70 + x) - x = 0$$

has a root α between 8 and 9.

The recurrence relation

$$x_{n+1} = 2\ln(70 + x_n)$$

with $x_0 = 8{\cdot}8$ can be used to find α.

Find and record the values of x_1, x_2, x_3. Write down the value of x_3 correct to four decimal places and prove that this value is the value of α correct to four decimal places. [7]

2008 Summer

5. *(a)* Show that $f(x) = \sin^{-1} x - 2x^{\frac{3}{2}} + 1$ has a stationary value when x satisfies

$$9x^3 - 9x + 1 = 0.$$ [4]

(b) Show that the equation

$$9x^3 - 9x + 1 = 0$$

has a root α between 0 and 0·2.

The recurrence relation

$$x_{n+1} = x_n^3 + \frac{1}{9}$$

with $x_0 = 0{\cdot}1$ can be used to find α. Find and record the values of x_1, x_2, x_3.

Write down the value of x_3 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

2009 Winter

4. *(a)* By sketching the graphs of $y = x^3$ and $y = 4 - x$, determine the number of real roots of the equation $x^3 + x - 4 = 0$. [3]

(b) **You may assume** that the equation $x^3 + x - 4 = 0$ has a root α between 1 and 2. The recurrence relation

$$x_{n+1} = (4 - x_n)^{\frac{1}{3}}$$

with $x_0 = 1{\cdot}4$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to four decimal places and prove that this value is the value of α correct to four decimal places. [5]

2009 Summer

4. *(a)* Show that $f(x) = (2x - 3)e^{2x} - 4x + 5$ has a stationary value when x satisfies

$$(x - 1)e^{2x} - 1 = 0.$$ [6]

(b) Show that the equation

$$(x - 1)e^{2x} - 1 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$x_{n+1} = 1 + e^{-2x_n}$$

with $x_0 = 1{\cdot}1$ may be used to find α. Find and record the values of x_1, x_2, x_3. Write down the value of x_3 correct to four decimal places and prove that this value is the value of α correct to four decimal places. [7]

2010 Winter

4. Show that the equation

$$2 - 10x + \sin x = 0$$

has a root α between 0 and $\frac{\pi}{8}$.

The recurrence relation

$$x_{n+1} = \frac{1}{10}\left(2 + \sin x_n\right),$$

with $x_0 = 0 \cdot 2$, can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

2010 Summer

4. Show that the equation

$$4x^3 - 2x - 5 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$x_{n+1} = \left(\frac{2x_n + 5}{4}\right)^{\frac{1}{3}},$$

with $x_0 = 1 \cdot 2$, may be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this value is the value of α correct to five decimal places. [7]

2011 Winter

4. **You may assume** that the equation $6x^4 + 7x - 3 = 0$ has a root α between 0 and 1.
The recurrence relation

$$x_{n+1} = \frac{3 - 6x_n^4}{7}$$

with $x_0 = 0 \cdot 4$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4.
Write down the value of x_4 correct to four decimal places and show this is the value of α correct to four decimal places. [5]

2011 Summer

4. *(a)* Show that $f(x) = 11\tan^{-1} 2x - 3x^2$ has a stationary value when x satisfies

$$12x^3 + 3x - 11 = 0 .$$ [3]

(b) **You may assume** that the equation $12x^3 + 3x - 11 = 0$ has a root α between 0 and 1.

The recurrence relation

$$x_{n+1} = \left(\frac{11 - 3x_n}{12}\right)^{\frac{1}{3}}$$

with $x_0 = 0{\cdot}9$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and show this is the value of α correct to five decimal places. [5]

2012 Winter

3. *(a)* A function is defined parametrically by

$$x = 3t^2,\ y = t^6 - 4t^3.$$

(i) Find $\frac{dy}{dx}$ in terms of t.

(ii) Given that $\frac{dy}{dx} = \frac{7}{2}$, show that $2t^4 - 4t - 7 = 0$. [5]

(b) Show that the equation

$$2t^4 - 4t - 7 = 0$$

has a root α between 1 and 2.

The recurrence relation

$$t_{n+1} = \left(\frac{4t_n + 7}{2}\right)^{\frac{1}{4}}$$

with $t_0 = 1{\cdot}6$ can be used to find α. Find and record the values of t_1, t_2, t_3, t_4. Write down the value of t_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

2012 Summer

4. Show that the equation

$$\cos x - 5x + 2 = 0$$

has a root α between 0 and $\frac{\pi}{4}$.

The recurrence relation

$$x_{n+1} = \frac{1}{5}(2 + \cos x_n)$$

with $x_0 = 0{\cdot}6$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

2013 Winter

4. *(a)* On the same diagram, sketch the graphs of $y = \ln x$ and $y = 11 - 2x$.
Deduce the number of roots of the equation

$$\ln x + 2x - 11 = 0.$$ [3]

(b) **You may assume** that the equation

$$\ln x + 2x - 11 = 0$$

has a root α between 4 and 5.
The recurrence relation

$$x_{n+1} = \frac{11 - \ln x_n}{2},$$

with $x_0 = 4{\cdot}7$, can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [5]

2013 Summer

8. **You may assume** that the equation

$$x^2 + e^x - 3 = 0$$

has a root α between -2 and -1.

The recurrence relation

$$x_{n+1} = -\left(3 - e^{x_n}\right)^{\frac{1}{2}}$$

with $x_0 = -1{\cdot}5$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [5]

2014 Winter

5. **You may assume** that the equation $x^3 + 7x^2 - 3 = 0$ has a root α between 0 and 1.
The recurrence relation

$$x_{n+1} = \sqrt{\frac{3}{x_n + 7}}$$

with $x_0 = 1$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4.

Write down the value of x_4 correct to five decimal places and show this is the value of α correct to five decimal places. [5]

2014 Summer

5. *(a)* Show that $f(x) = \ln(3x^2 - 2x - 1) - 4x^2$ has a stationary value when x satisfies

$$12x^3 - 8x^2 - 7x + 1 = 0.$$ [4]

(b) **You may assume** that the equation $12x^3 - 8x^2 - 7x + 1 = 0$ has a root α between -1 and 0.
The recurrence relation

$$x_{n+1} = \left(\frac{8x_n^2 + 7x_n - 1}{12}\right)^{\frac{1}{3}}$$

with $x_0 = -0{\cdot}6$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to four decimal places and show this is the value of α correct to four decimal places. [5]

2015

5. *(a)* On the same diagram, sketch the graphs of $y = \cos^{-1} x$ and $y = 5x - 1$. [2]

(b) **You may assume** that the equation

$$\cos^{-1} x - 5x + 1 = 0$$

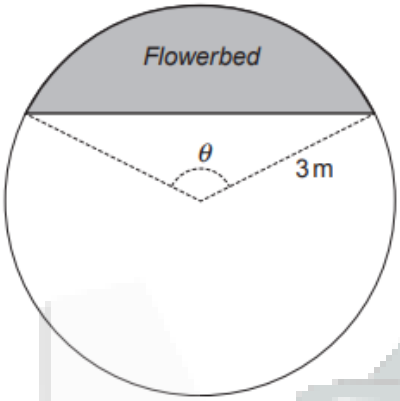
has a root α between $0{\cdot}4$ and $0{\cdot}5$.
The recurrence relation

$$x_{n+1} = \frac{1}{5}(1 + \cos^{-1} x_n)$$

with $x_0 = 0{\cdot}4$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to four decimal places and prove that this is the value of α correct to four decimal places. [5]

2016

5. The diagram shows a circular garden plot of radius 3 m. Alun wants to use a minor segment of the plot as a flowerbed and has a 13·5 m length of edging, all of which he intends to use to form the perimeter of the shaded area below. The angle subtended at the centre of the circular plot is denoted by θ radians.



(a) Show that θ satisfies the equation

$$\theta + 2\sin\left(\frac{\theta}{2}\right) = 4{\cdot}5.$$ [3]

(b) Alun believes that the value of θ will turn out to be approximately 2·5.
Starting with $\theta_0 = 2{\cdot}5$, use the recurrence relation

$$\theta_{n+1} = 4{\cdot}5 - 2\sin\left(\frac{\theta_n}{2}\right)$$

to find the values of θ_1, θ_2, θ_3. Write down the value of θ_3 correct to two decimal places and prove that this is the value of θ correct to two decimal places. [5]

2017

4. A large tank in the form of a cuboid is used to store water. The width of the tank is denoted by x m. The length of the tank is 4 m **greater** than its width, whilst the height of the tank is 2 m **less** than its width. The volume of the tank is 150 m^3.

(a) (i) Show that $x^3 + 2x^2 - 8x - 150 = 0$.

(ii) Show that $5 < x < 6$. [4]

(b) The recurrence relation

$$x_{n+1} = \left(150 + 8x_n - 2x_n^2\right)^{\frac{1}{3}},$$

with $x_0 = 6$, can be used to find the value of x. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to two decimal places and prove that this is the value of x correct to two decimal places. [5]