

C3 Parametric Differentiation Answers

Specimen

4. (a)  $3y^2 \frac{dy}{dx} - 2x^2 y \frac{dy}{dx} - 2xy^2 = 2x + 3$

B1  $\left(3y^2 \frac{dy}{dx}\right)$

M1  $\left(-2x^2 y \frac{dy}{dx} \pm 2xy^2\right)$

A1

$\therefore \frac{dy}{dx} = \frac{2x + 3 + 2xy^2}{3y^2 - 2x^2 y}$

A1 (all correct)

(b)  $\frac{dy}{dx} = \frac{2t}{3t^2} = \frac{2}{3t}$

M1  $\left(\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}\right)$

A1

$\therefore \frac{d^2 y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

M1 (correct formula)

$= -\frac{\frac{2}{3t^2}}{3t^2} = -\frac{2}{9t^4}$

A1, A1 (convincing)

2005 Summer

4. (a)  $2x + 2x \frac{dy}{dx} + 2y + 6y \frac{dy}{dx} = 0$

B1  $\left(2x \frac{dy}{dx} + 2y\right)$

B1  $\left(6y \frac{dy}{dx}\right)$

B1 (all correct)

$\frac{dy}{dx} = -\frac{x+y}{x+3y}$  (o.e)

(b)  $\frac{dy}{dx} = \frac{6t}{8t^3} = \frac{3}{4t^2}$

M1  $\left(\frac{\dot{y}}{\dot{x}}\right)$  A1 (correct)

$\frac{d^2 y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{-\frac{3}{2t^3}}{8t^3}$

M1 (correct formula)

$= -\frac{3}{16t^6}$

A1 (F.T one slip in  $\frac{dy}{dx}$ )

2006 Winter

3. (a)  $4y^3 \frac{dy}{dx} + x^3 \frac{dy}{dx} + 3x^2 y = 2x + 4$

B1 ( $4y^3 \frac{dy}{dx}$ )

B1 ( $x^3 \frac{dy}{dx} + 3x^2 y$ )

B1 ( $2x + 4$ )

$$4 \frac{dy}{dx} + 8 \frac{dy}{dx} + 12 = 8$$

$$\frac{dy}{dx} = -\frac{4}{12} \quad (\text{o.e.})$$

B1 (C.A.O.)

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(b) (i)  $\frac{dy}{dx} = \frac{12t^3}{6t^2} \quad (\text{o.e.})$

M1 A1

(ii)  $\frac{d^2 y}{dx^2} = \frac{\frac{d}{dt} \left( \frac{dy}{dx} \right)}{\frac{dx}{dt}} = \frac{2}{6t^2} \quad (\text{o.e.})$

M1 (use of correct formula)

A1 (F.T. one slip)

8

2006 Summer

3. (a)  $\frac{dy}{dx} = \frac{2 \cos 2t}{-\sin t}$

M1 (attempt to use  $\frac{dy}{dx} = \frac{\dot{y}}{x}$ ),

B1 ( $-\sin t$ )

B1 ( $k \cos 2t, k = 1, 2, -2, \frac{1}{2}$ )

A1 ( $\left( \frac{2 \cos 2t}{-\sin t}, \text{C.A.O.} \right)$ )

(b)  $4x^3 + 2x^2 \frac{dy}{dx} + 4xy + 2y \frac{dy}{dx} = 0$

B1 ( $2x^2 + 4xy$ )

B1 ( $2y \frac{dy}{dx}$ )

B1 ( $4x^3, 0$ )

B1 (C.A.O.)

$$\frac{dy}{dx} = \frac{(4x^3 + 4xy)}{2x^2 + 2y}$$

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2007 Winter

6. (a)  $3x^2 + x^2 \frac{dy}{dx} + 2xy + 4y^3 \frac{dy}{dx} = 0$

B1 ( $x^2 \frac{dy}{dx} + 2xy$ )

B1 ( $4y^3 \frac{dy}{dx}$ )

$$\frac{dy}{dx} = -\frac{3x^2 + 2xy}{x^2 + 4y^3}$$

B1 (all correct, for final result C.A.O.)

(b) (i)  $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{3t^2} \left( = \frac{2}{3t} \right)$

M1 ( $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$ )

A1 (one differentiation)  
A1 (other differentiation)

$$\frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right) = \frac{-2}{3t^2}$$

M1 (correct formula)

A1 (correct differentiation)  
F.T. one slip for differentiation of equivalent difficulty

$$= -\frac{2}{9t^4}$$

(o.e.)

A1 (C.A.O.)

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2007 Summer

3. (a) (i)  $\frac{dy}{dx} = \frac{5t^4 + 20t^2}{10t}$

M1 (attempt to use  $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$ )

A1 A1

(ii)  $\frac{5t^4 + 20t^2}{10t} = 1$   
 $\frac{t^3 + 4t}{2} = 1$

M1 (use of equation and attempt to simplify)

$$t^3 + 4t - 2 = 0$$

A1 (convincing)

2008 Winter

3. (a)  $\frac{dy}{dx} = \frac{2e^{2t}}{4t^3}$

M1 ( $\dot{y} = ke^{2t}$ ,  $k = 1$  or  $2$  or  $2e^{2t} + 5$ )

A1 ( $2e^{2t}$ )

M1 ( $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$ ), A1 (all correct C.A.O.)

(b)  $4x^3 + \cos y \frac{dy}{dx} + 2xy^3 + 3x^2y^2 \frac{dy}{dx} = 0$

B1 ( $\cos y \frac{dy}{dx}$ )

B1 ( $2xy^3 + 3x^2 \frac{dy}{dx}$ )

B1 (F.T.  $\frac{d}{dy}(\sin y) = -\cos y \frac{dy}{dx}$ )

$$\frac{dy}{dx} = -\frac{4x^3 + 2xy^3}{\cos y + 3x^2y^2}$$

2008 Summer

4. (a)  $\frac{dy}{dt} = 2e^{2t}$

M1 ( $ke^{2t}$ ,  $k = \frac{1}{2}, 1, 2$ )

A1 ( $k = 2$ )

$$\frac{dy}{dt} = \frac{1}{t}$$

B1

$$\frac{dy}{dx} = 2te^{2t}$$

(o.e.)

A1 (C.A.O.)

(b)  $\frac{d}{dt}\left(\frac{dy}{dx}\right) = 2te^{2t} + 4te^{2t}$

M1 ( $f(t)e^{2t} + 2tg(t)$ )

A1 ( $f(t) = 2$ ,  
 $g(t) = 2e^{2t}$ )

$$\frac{d^2y}{dx^2} = 2te^{2t}(1+2t) \quad \text{(o.e.)}$$

M1 (use of

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) + \frac{dx}{dt}$$

A1 (F. T candidates  $\frac{d}{dt}\left(\frac{dy}{dx}\right)$ )

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2009 Winter

3. (a)  $2x + 3x \frac{dy}{dx} + 3y + 4y \frac{dy}{dx} - 2 = 0$   
 $2 + 3 \frac{dy}{dx} + 6 + 8 \frac{dy}{dx} - 2 = 0$

$$\frac{dy}{dx} = -\frac{6}{11}$$

(b)  $\frac{dy}{dx} = \frac{8e^{2t} + 3e^t}{2e^t}$

$$\frac{8e^{2t} + 3e^t}{2e^t} = 6$$

$$8e^t = 9$$

$$t = \ln\left(\frac{9}{8}\right) \approx 0.118$$

2009 Summer

3. (a)  $3x^2 + 2y \frac{dy}{dx} + \tan 2y + 2x \sec^2 2y \frac{dy}{dx} = 0$

$$\frac{dy}{dx} = -\frac{3x^2 + \tan 2y}{2y + 2x \sec^2 2y}$$

B1  $\left(3x \frac{dy}{dx} + 3y\right) (o.e)$

B1  $\left(4y \frac{dy}{dx}\right) (o.e)$

B1 (correct diff<sup>n</sup> of  $x^2$ ,  $-2x$  and  $13$ )

B1 (F.T. one slip)

M1

B1 ( numerator  $ke^{2t} + 3e^t$ ,  $k=4,8$  )

B1 ( $k=8$ )

B1 (denominator)

M1

M1

A1 (C.A.O)

(11)

$\left(2y \frac{dy}{dx}\right)$  B1

$\left(\tan 2y + k \sec^2 2y \frac{dy}{dx}; k=1, 2\right)$  B1

( $k=2$ ) B1

(All correct. F.T. for last B1 if first two Bs gained) B1

(b) (i)  $\frac{dy}{dt} = \frac{(3+2t)(4) - (1+4t)(2)}{(3+2t)^2}$   $\left( \frac{(3+2t)f(t) - (1+4t)g(t)}{(3+2t)^2} \right)$  M1  
 $[f(t) = 4, g(t) = 2]$  A1  
 (Give additional mark for simplification here, see last A1 of part (ii))

(ii)  $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$  (attempt to use  $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$ ) M1  
 $= \frac{(3+2t)(4) - (1+4t)(2)}{(3+2t)^2} \cdot \frac{1}{(3+2t)}$  A1  
 $= \frac{10}{(3+2t)^3}$  (This may have been given in (i).) A1  
 (Penalise faulty simplification on last line)

2010 Winter

3. (a)  $\frac{d(y^3)}{dx} = 3y^2 \frac{dy}{dx}$  B1  
 $\frac{d(2x^3y)}{dx} = 2x^3 \frac{dy}{dx} + 6x^2y$  B1  
 $\frac{d(3x^2 + 4x - 3)}{dx} = 6x + 4$  B1  
 $x = 2, y = 1 \Rightarrow \frac{dy}{dx} = \frac{-8}{19}$  (c.a.o.) B1

(b) (i)  $\frac{dx}{dt} = 6t, \frac{dy}{dt} = 12t^2 + 6t^5$  (all three terms correct) B2  
 (one term correct) B1  
 Use of  $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$  M1  
 $\frac{dy}{dx} = 2t + t^4$  (c.a.o.) A1

(ii)  $\frac{d}{dt} \left[ \frac{dy}{dx} \right] = 2 + 4t^3$  (f.t. candidate's expression for  $\frac{dy}{dx}$ ) B1  
 Use of  $\frac{d^2y}{dx^2} = \frac{d}{dt} \left[ \frac{dy}{dx} \right] \div \frac{dx}{dt}$  M1  
 $\frac{d^2y}{dx^2} = \frac{1 + 2t^3}{3t}$  (c.a.o.) A1

2010 Summer

3. (a)  $\frac{d(y^4)}{dx} = 4y^3 \frac{dy}{dx}$  B1  
 $\frac{d(4x^2y)}{dx} = 4x^2 \frac{dy}{dx} + 8xy$  B1  
 $\frac{d(3x^3 - 5x)}{dx} = 9x^2 - 5$  B1  
 $\frac{dy}{dx} = \frac{9x^2 - 5 - 8xy}{4y^3 + 4x^2}$  (c.a.o.) B1

(b)  $\frac{dx}{dt} = 4 - 2 \sin 2t$ , B1  
 $\frac{dy}{dt} = 3 \cos 3t$  B1  
Use of  $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$  M1  
Substituting  $\frac{\pi}{12}$  for  $t$  in expression for  $\frac{dy}{dx}$  m1  
 $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$  A1

2011 Winter

3. (a)  $\frac{d(2y^2)}{dx} = 4y \frac{dy}{dx}$  B1  
 $\frac{d(3x^2y)}{dx} = 3x^2 \frac{dy}{dx} + 6xy$  B1  
 $\frac{d(x^4)}{dx} = 4x^3$ ,  $\frac{d(15)}{dx} = 0$  B1  
 $\frac{dy}{dx} = \frac{4x^3 + 6xy}{4y - 3x^2}$  (c.a.o.) B1

(b) (i) Differentiating  $\ln t$  and  $t^3 - 7t$  with respect to  $t$ , at least one correct M1

candidate's  $x$ -derivative =  $\frac{1}{t}$ ,

candidate's  $y$ -derivative =  $3t^2 - 7$  (both values) A1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$  M1

$\frac{dy}{dx} = 3t^3 - 7t$  (c.a.o.) A1

(ii)  $\frac{d}{dt} \left( \frac{dy}{dx} \right) = 9t^2 - 7$  (f.t. candidate's expression for  $\frac{dy}{dx}$ ) B1

Use of  $\frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right) \div \text{candidate's } x\text{-derivative}$  M1

$\frac{d^2y}{dx^2} = \frac{9t^2 - 7}{3t}$  (f.t. one slip) A1

When  $t = \frac{1}{3}$ ,  $\frac{d^2y}{dx^2} = -2$  (c.a.o.) A1

2011 Summer

3. (a)  $\frac{d(2x^3)}{dx} = 6x^2$ ,  $\frac{d(2x)}{dx} = 2$ ,  $\frac{d(25)}{dx} = 0$  B1

$\frac{d(x^2 \cos y)}{dx} = x^2(-\sin y) \frac{dy}{dx} + 2x(\cos y)$  B1

$\frac{d(y^4)}{dx} = 4y^3 \frac{dy}{dx}$  B1

$\frac{dy}{dx} = \frac{6x^2 + 2x \cos y + 2}{x^2 \sin y - 4y^3}$  (c.a.o.) B1

(b) (i) candidate's  $x$ -derivative =  $3t^2$   
 candidate's  $y$ -derivative =  $4t + 20t^3$   
 (one term correct B1, all three terms correct B2)

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$  M1

$\frac{dy}{dx} = \frac{4 + 20t^2}{3t}$  (c.a.o.) A1

(ii)  $\frac{dy}{dx} = 5 \Rightarrow 20t^2 - 15t + 4 = 0$   
 (f.t. candidate's expression for  $\frac{dy}{dx}$  from (i)) B1

Considering  $b^2 - 4ac$  for candidate's quadratic M1

$b^2 - 4ac = 225 - 320 < 0$  and hence no such real value of  $t$  exists  
 (f.t. candidate's quadratic) A1



2012 Winter

3. (a)  $\frac{d}{dx}(x^3) = 3x^2$   $\frac{d}{dx}(-3x-2) = -3$  B1  
 $\frac{d}{dx}(-4x^2y) = -4x^2\frac{dy}{dx} - 8xy$  B1  
 $\frac{d}{dx}(2y^3) = 6y^2\frac{dy}{dx}$  B1  
 $x = 3, y = 1 \Rightarrow \frac{dy}{dx} = \frac{6}{42} = \frac{1}{7}$  (c.a.o.) B1

(b) (i) Differentiating  $\sin at$  and  $\cos at$  with respect to  $t$ , at least one correct M1  
 candidate's  $x$ -derivative =  $a \cos at$ ,  
 candidate's  $y$ -derivative =  $-a \sin at$  (both values) A1  
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$  M1  
 $\frac{dy}{dx} = -\tan at$  (c.a.o.) A1

(ii)  $\frac{d}{dt}\left(\frac{dy}{dx}\right) = -a \sec^2 at$  (f.t. candidate's expression for  $\frac{dy}{dx}$ ) B1  
 Use of  $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \text{candidate's } x\text{-derivative}$  M1  
 $\frac{d^2y}{dx^2} = -\sec^3 at$  (c.a.o.) A1

2012 Summer

3. (a)  $\frac{d}{dx}(x^3) = 3x^2$   $\frac{d}{dx}(-3x-2) = -3$  B1  
 $\frac{d}{dx}(-4x^2y) = -4x^2\frac{dy}{dx} - 8xy$  B1  
 $\frac{d}{dx}(2y^3) = 6y^2\frac{dy}{dx}$  B1  
 $x = 3, y = 1 \Rightarrow \frac{dy}{dx} = \frac{6}{42} = \frac{1}{7}$  (c.a.o.) B1

- (b) (i) Differentiating  $\sin at$  and  $\cos at$  with respect to  $t$ , at least one correct M1  
 candidate's  $x$ -derivative =  $a \cos at$ ,  
 candidate's  $y$ -derivative =  $-a \sin at$  (both values) A1  
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$  M1  
 $\frac{dy}{dx} = -\tan at$  (c.a.o.) A1
- (ii)  $\frac{d}{dt} \left( \frac{dy}{dx} \right) = -a \sec^2 at$  (f.t. candidate's expression for  $\frac{dy}{dx}$ ) B1  
 Use of  $\frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right) \div \frac{dx}{dt}$  candidate's  $x$ -derivative M1  
 $\frac{d^2y}{dx^2} = -\sec^3 at$  (c.a.o.) A1

2013 Winter

3. (a)  $\frac{d(2y^3)}{dx} = 6y^2 \frac{dy}{dx}$  B1  
 $\frac{d(5x^4y)}{dx} = 5x^4 \frac{dy}{dx} + 20x^3y$  B1  
 $\frac{d(x^3)}{dx} = 3x^2, \frac{d(7)}{dx} = 0$  B1  
 $\frac{dy}{dx} = \frac{20x^3y + 3x^2}{6y^2 - 5x^4}$  (o.e.) (c.a.o.) B1

- (b) (i) candidate's  $x$ -derivative =  $3t^2$  B1  
 candidate's  $y$ -derivative =  $4t^3 + 35t^4$  B1  
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$  M1  
 $\frac{dy}{dx} = \frac{4t^3 + 35t^4}{3t^2}$  (c.a.o.) A1

- (ii)  $\frac{d}{dt} \left( \frac{dy}{dx} \right) = \frac{4 + 70t}{3}$  (o.e.) B1

Use of  $\frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right) \div \frac{dx}{dt}$   
 (f.t. candidate's expression for  $\frac{dy}{dx}$ ) M1  
 $\frac{d^2y}{dx^2} = \frac{4 + 70t}{9t^2}$  (o.e.) A1

- (iii) An attempt to solve  $t^3 - 5 = 3$  and substitution of answer in candidate's expression for  $\frac{d^2y}{dx^2}$  M1

$\frac{d^2y}{dx^2} = 4$  (c.a.o.) A1

2013 Summer

3. (a) Use of product formula yielding  $x^3 \times 2y \times \frac{dy}{dx} + 3x^2 \times y^2$  B1 B1  
 $\frac{dy}{dx} = -\frac{3x^2y^2}{2x^3y}$  (c.a.o.) B1
- (b) (i) Putting candidate's expression for  $\frac{dy}{dx} = 3$  and an attempt to simplify M1  
 $-\frac{3a^2b^2}{2a^3b} = 3 \Rightarrow b = -2a$  (convincing) A1
- (ii) Substituting  $a$  for  $x$  and  $-2a$  for  $y$  in the equation for  $C$  M1  
 $a = 2, b = -4$  A1

2014 Winter

4. (a)  $\frac{dx}{dt} = 6t^2$  B1
- (b)  $\frac{d}{dt} \left( \frac{dy}{dx} \right) = 2 + 12t^2$  B1  
 Use of  $\frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right) \div \frac{dx}{dt}$  M1  
 $\frac{d^2y}{dx^2} = \frac{2 + 12t^2}{6t^2}$  (c.a.o.) A1  
 $\frac{d^2y}{dx^2} = 2 \Rightarrow 2 + 12t^2 = 12t^2 (\Rightarrow 2 = 0) \Rightarrow$  no such  $t$  exists E1
- (c) Use of  $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$  M1  
 $\frac{dy}{dt} = 12t^3 + 24t^5$  (f.t. candidate's expression for  $\frac{dx}{dt}$ ) A1  
 Use of a valid method of integration to find  $y$  m1  
 $y = 3t^4 + 4t^6 (+ c)$  (f.t. one error in candidate's  $\frac{dy}{dt}$ ) A1  
 $y = 3t^4 + 4t^6 + 3$  (c.a.o.) A1

3. (a)  $\frac{d(y^4)}{dx} = 4y^3 \frac{dy}{dx}$  B1  
 $\frac{d(8xy^2)}{dx} = (8x)(2y)\frac{dy}{dx} + 8y^2$  B1  
 $\frac{d(2x^2)}{dx} = 4x, \frac{d(9)}{dx} = 0$  B1  
 $\frac{dy}{dx} = \frac{x - 2y^2}{y^3 + 4xy}$  (convincing) (c.a.o.) B1

(b)  $\frac{dy}{dx} = 0 \Rightarrow x = 2y^2$  B1  
 dx  
 Substitute  $2y^2$  for  $x$  in equation of C M1  
 $9y^4 + 9 = 0$  (o.e.) (c.a.o.) A1  
 $9y^4 + 9 > 0$  for any real  $y$  (o.e.) and thus no such point exists A1

4. (a) candidate's  $x$ -derivative =  $\frac{1}{1+t^2}$  B1  
 candidate's  $y$ -derivative =  $\frac{1}{t}$  B1  
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$  M1  
 $\frac{dy}{dx} = \frac{1+t^2}{t}$  A1

(b)  $\frac{d}{dt} \left( \frac{dy}{dx} \right) = -t^{-2} + 1$  (o.e.) B1  
 Use of  $\frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right) \div \text{candidate's } x\text{-derivative}$  M1  
 $\frac{d^2y}{dx^2} = (-t^{-2} + 1)(1+t^2)$  (o.e.) (f.t. one slip) A1  
 $\frac{d^2y}{dx^2} = 0 \Rightarrow t = 1$  (c.a.o.) A1  
 $\frac{d^2y}{dx^2} = 0 \Rightarrow x = \frac{\pi}{4}$  (f.t. candidate's derived value for  $t$ ) A1

2016

4. (a) candidate's  $x$ -derivative =  $12 \cos 3t$  B1  
 candidate's  $y$ -derivative =  $-6 \sin 3t$  B1  
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$  M1  
 $\frac{dy}{dx} = -\frac{1}{2} \tan 3t$  (c.a.o.) A1

(b) (i)  $\frac{d}{dt} \left( \frac{dy}{dx} \right) = -\frac{3}{2} \sec^2 3t$  (f.t.  $\frac{dy}{dx} = k \tan 3t$  or  $k \frac{\sin 3t}{\cos 3t}$  only) B1

Use of  $\frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right) \div \frac{dx}{dt}$  ÷ candidate's  $x$ -derivative M1

$\frac{d^2y}{dx^2} = -\frac{1}{8} \sec^3 3t$  or  $-\frac{1}{8 \cos^3 3t}$  (c.a.o.) A1

(ii)  $\frac{d^2y}{dx^2} = -\frac{1}{y^3}$  (f.t.  $\frac{d^2y}{dx^2} = m \sec^3 3t$  or  $\frac{m}{\cos^3 3t}$  only) B1

2017

3. (a)  $\frac{d(2y^3)}{dx} = 6y^2 \frac{dy}{dx}$  B1  
 $\frac{d(-3x^2y)}{dx} = -3x^2 \frac{dy}{dx} - 6xy$  B1  
 $\frac{d(x^4)}{dx} = 4x^3, \frac{d(-4x)}{dx} = -4, \frac{d(7)}{dx} = 0$  B1  
 $\frac{dy}{dx} = \frac{4 - 4x^3 + 6xy}{6y^2 - 3x^2}$  (o.e.) (c.a.o.) B1

(b) (i) candidate's  $x$ -derivative =  $7 + 4t$  B1  
 candidate's  $y$ -derivative =  $\frac{(7 + 4t)r - (4 + 3t)m}{(7 + 4t)^2}$

where  $r, m$  are integers M1

candidate's  $y$ -derivative =  $\frac{(7 + 4t)3 - (4 + 3t)4}{(7 + 4t)^2}$  A1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$  M1

$\frac{dy}{dx} = \frac{5}{(7 + 4t)^3}$  (c.a.o.) A1

(ii)  $\frac{d}{dt} \left( \frac{dy}{dx} \right) = \frac{-3 \times 5 \times 4}{(7 + 4t)^4}$  (o.e.) B1  
 (f.t. candidate's expression of correct given form for  $\frac{dy}{dx}$ )

Use of  $\frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right) \div \frac{dx}{dt}$  M1  
 (f.t. candidate's expression for  $\frac{d}{dt} \left( \frac{dy}{dx} \right)$ )

$\frac{d^2y}{dx^2} = \frac{-60}{(7 + 4t)^5}$  (c.a.o.) A1