C3 Parametric Differentiation Questions

Specimen

4. *(a)* Given that

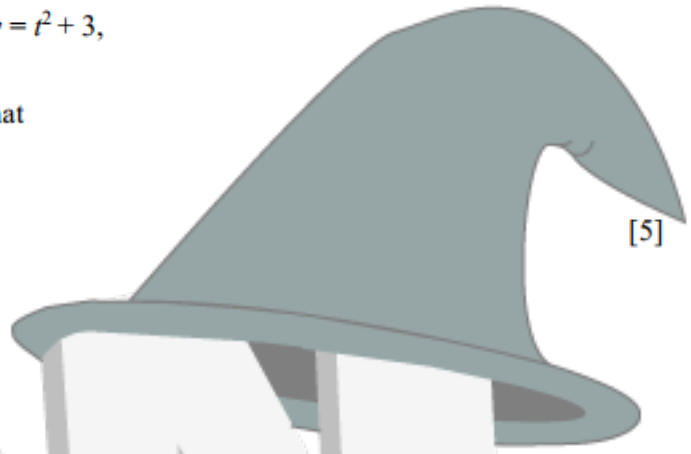
$$y^3 - x^2y^2 = x^2 + 3x + 1,$$

find $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that $x = t^3 + 2,\ y = t^2 + 3,$

find $\frac{dy}{dx}$ and show that

$$\frac{d^2y}{dx^2} = -\frac{2}{9t^4}.$$ [5]



2005 Summer

4. *(a)* A function is defined implicitly by

$$x^2 + 2xy + 3y^2 = 12.$$

Find $\frac{dy}{dx}$ in terms of x and y. [3]

(b) Another function is defined parametrically by $x = 2t^4,\ y = 3t^2$.

(i) Find $\frac{dy}{dx}$ in term of t.

(ii) Find $\frac{d^2y}{dx^2}$ in terms of t. [4]

2006 Winter

3. *(a)* The curve C is defined by

$$y^4 + x^3y = x^2 + 4x - 3.$$

Find the value of $\frac{dy}{dx}$ at the point (2, 1) [4]

(b) Given that $x = 2t^3$, $y = 3t^4$, find, in terms of t,

(i) $\frac{dy}{dx}$,

(ii) $\frac{d^2y}{dx^2}$. [4]

2006 Summer

3. *(a)* Given that $x = \cos t$, $y = \sin 2t$, find $\frac{dy}{dx}$ in terms of t. [4]

(b) Given that

$$x^4 + 2x^2y + y^2 = 21,$$

find $\frac{dy}{dx}$ in terms of x and y. [4]

2007 Winter

6. *(a)* Given that $x^3 + x^2y + y^4 = 17$, find $\frac{dy}{dx}$ in terms of x and y. [3]

(b) Given that $x = t^3$, $y = t^2 + 1$, find, in terms of t,

(i) $\frac{dy}{dx}$,

(ii) $\frac{d^2y}{dx^2}$. [6]

2007 Summer

3. *(a)* A function is defined parametrically by $x = 5t^2$, $y = t^5 + \frac{20t^3}{3}$.

(i) Find $\frac{dy}{dx}$ in terms of t.

(ii) Given that $\frac{dy}{dx} = 1$, show that $t^3 + 4t - 2 = 0$. [5]

2008 Winter

3. *(a)* Given that $x = t^4 + 1$, $y = e^{2t} + 5$, find $\frac{dy}{dx}$ in terms of t. [4]

(b) Given that $x^4 + \sin y + x^2y^3 = 9$, find $\frac{dy}{dx}$ in terms of x and y. [3]

2008 Summer

4. Given that $x = \ln t$, $y = e^{2t}$,

(a) show that $\frac{dy}{dx} = 2te^{2t}$, [4]

(b) find $\frac{d^2y}{dx^2}$ in terms of t, simplifying your answer. [4]

2009 Winter

3. *(a)* Given that $x^2 + 3xy + 2y^2 - 2x = 13$, find the value of $\frac{dy}{dx}$ at the point $(1, 2)$. [4]

(b) Given that $x = 2e^t + 6$, $y = 4e^{2t} + 3e^t + 1$, find the value of t when $\frac{dy}{dx} = 6$, giving your answer correct to three decimal places. [7]

2009 Summer

3. *(a)* Given that

$$x^3 + y^2 + x \tan 2y = 8,$$

find $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that $x = 3t + t^2, y = \frac{1+4t}{3+2t}$, find

(i) $\frac{dy}{dt}$,

(ii) $\frac{dy}{dx}$, simplifying your answer as much as possible. [5]

2010 Winter

3. *(a)* The curve C is defined by

$$y^3 + 2x^3y = 3x^2 + 4x - 3.$$

Find the value of $\frac{dy}{dx}$ at the point (2, 1). [4]

(b) Given that $x = 3t^2, y = 4t^3 + t^6$, find, in terms of t,

(i) $\frac{dy}{dx}$,

(ii) $\frac{d^2y}{dx^2}$.

Simplify your answers. [7]

2010 Summer

3. *(a)* Given that

$$y^4 + 4x^2y = 3x^3 - 5x,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that $x = 4t + \cos 2t$, $y = \sin 3t$, show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ when $t = \frac{\pi}{12}$. [5]

2011 Winter

3. *(a)* Given that

$$x^4 + 3x^2y - 2y^2 = 15,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that $x = \ln t$, $y = t^3 - 7t$,

(i) find an expression for $\frac{dy}{dx}$ in terms of t,

(ii) find the value of $\frac{d^2y}{dx^2}$ when $t = \frac{1}{3}$. [8]

2011 Summer

3. *(a)* Given that

$$2x^3 + x^2 \cos y + y^4 + 2x - 25 = 0,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that

$$x = t^3, \quad y = 2t^2 + 5t^4,$$

(i) find and simplify an expression for $\frac{dy}{dx}$ in terms of t,

(ii) show that there is no real value of t for which $\frac{dy}{dx} = 5$. [7]

2012 Winter

3. *(a)* The curve C is defined by

$$x^3 - 4x^2y = 2y^3 - 3x - 2.$$

Find the value of $\frac{dy}{dx}$ at the point (3, 1). [4]

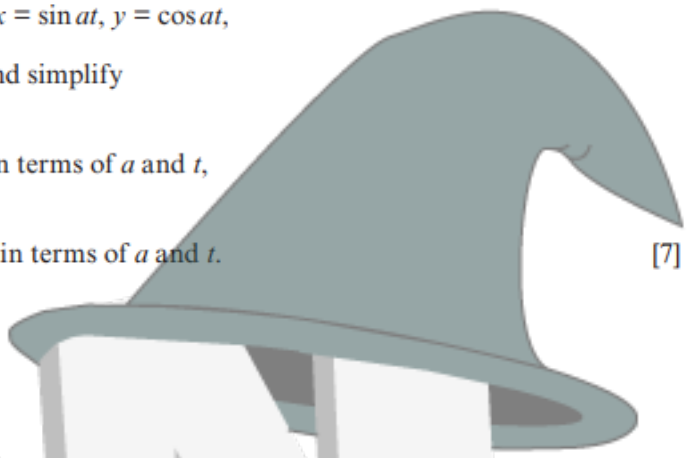
(b) Given that

$$x = \sin at, \ y = \cos at,$$

where a is a constant, find and simplify

(i) an expression for $\frac{dy}{dx}$ in terms of a and t,

(ii) an expression for $\frac{d^2y}{dx^2}$ in terms of a and t. [7]



2012 Summer

3. *(a)* The curve C is defined by

$$x^3 - 4x^2y = 2y^3 - 3x - 2.$$

Find the value of $\frac{dy}{dx}$ at the point (3, 1). [4]

(b) Given that

$$x = \sin at, \ y = \cos at,$$

where a is a constant, find and simplify

(i) an expression for $\frac{dy}{dx}$ in terms of a and t,

(ii) an expression for $\frac{d^2y}{dx^2}$ in terms of a and t. [7]

2013 Winter

3. *(a)* Given that

$$x^3 + 5x^4y - 2y^3 + 7 = 0,$$

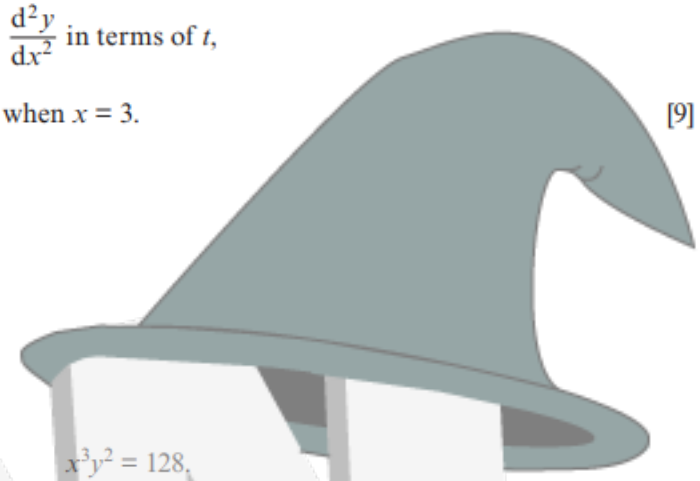
find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that $x = t^3 - 5$, $y = t^4 + 7t^5$,

(i) find an expression for $\frac{dy}{dx}$ in terms of t,

(ii) find an expression for $\frac{d^2y}{dx^2}$ in terms of t,

(iii) find the value of $\frac{d^2y}{dx^2}$ when $x = 3$. [9]



2013 Summer

3. The curve C is defined by

$$x^3y^2 = 128.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y. [3]

The point P lies on C and has coordinates (a, b).

(b) Given that the value of $\frac{dy}{dx}$ at the point P is 3,

(i) show that $b = -2a$,

(ii) find the value of a and the value of b. [4]

2014 Winter

4. The variables x and y are defined parametrically in terms of the variable t. It is known that $x = 2t^3$ and that $\frac{dy}{dx} = 2t + 4t^3$.

(a) Find an expression for $\frac{dx}{dt}$ in terms of t. [1]

(b) Find an expression for $\frac{d^2y}{dx^2}$ in terms of t and hence show there is no value of t for which

$$\frac{d^2y}{dx^2} = 2.$$ [4]

(c) Given that $y = 10$ when $t = 1$, find an expression for y in terms of t. [5]

2014 Summer

3. The curve C is defined by

$$y^4 - 2x^2 + 8xy^2 + 9 = 0.$$

(a) Show that $\frac{dy}{dx} = \frac{x - 2y^2}{y^3 + 4xy}$. [4]

(b) Show that there is no point on C at which $\frac{dy}{dx} = 0$. [4]

2015

4. Given that $x = \tan^{-1} t$, $y = \ln t$, where $t > 0$,

(a) find an expression for $\frac{dy}{dx}$ in terms of t, [4]

(b) find the value of x for which $\frac{d^2y}{dx^2} = 0$. [5]

2016

4. A function is defined parametrically by

$$x = 4\sin 3t, \ y = 2\cos 3t.$$

(a) Find and simplify an expression for $\frac{dy}{dx}$ in terms of t. [4]

(b) Find and simplify an expression for $\frac{d^2y}{dx^2}$

(i) in terms of t,

(ii) in terms of y. [4]

2017

3. *(a)* Given that

$$x^4 - 3x^2y + 2y^3 - 4x = 7,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that $x = 7t + 2t^2$, $y = \frac{4+3t}{7+4t}$,

(i) show that $\frac{dy}{dx} = \frac{k}{(7+4t)^n}$,

where the values of the constants k and n are to be found,

(ii) find a similar expression for $\frac{d^2y}{dx^2}$. [8]