# C4 Binomial Expansion Answers

## Specimen

**1.** $1+\left(-\frac{1}{2}\right)(2x)+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(2x)^2}{1.2}$

$+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(2x)^3}{1.2.3}+\ldots$

$=1-x+\frac{3}{2}x^2-\frac{5}{2}x^3+\ldots$ B1 $(1-x)$ B1 $\left(\frac{3}{2}x^2\right)$ B1 $\left(-\frac{5}{2}x^3\right)$

$\dfrac{(1-x)^2}{(1+2x)^{\frac{1}{2}}}=(1-2x+x^2)(1-x+\frac{3}{2}x^2-\frac{5}{2}x^3+\ldots)$

$=1-x+\frac{3}{2}x^2-\frac{5}{2}x^3+\ldots$

$-2x+2x^2-3x^3+\ldots$

$+x^2-x^3+\ldots$

$=1-3x+\frac{9}{2}x^2-\frac{13}{2}x^3+\ldots$ B1 $(1-3x)$ B1 $\left(\frac{9}{2}x^2\right)$ B1 $\left(-\frac{13}{2}x^3\right)$

Valid for $|x|<\frac{1}{2}$ B1

## 2005 Summer

**2.** $(1-2x)^{-\frac{1}{2}}=1+\left(-\dfrac{1}{2}\right)(-2x)+\left(-\dfrac{1}{2}\right)\left(-\dfrac{3}{2}\right)\dfrac{1}{2!}(-2x)^2+\ldots\ldots\ldots$

$=1+x+\dfrac{3}{2}x^2+\ldots\ldots\ldots$ B1 $(1+x)$ B1 $\left(\dfrac{3}{2}x^2\right)$

Expansion valid for $|x|<\dfrac{1}{2}$ B1

$\left(1-\dfrac{1}{4}\right)^{-\frac{1}{2}}\approx 1+\dfrac{1}{8}+\dfrac{3}{128}=\dfrac{147}{128}$ B1 (F.T one slip)

$\left(\dfrac{3}{4}\right)^{\frac{1}{2}}=\dfrac{128}{147}$

$\therefore \sqrt{3}\approx\dfrac{256}{147}$ B1 (C.A.O)

[5]

**2006 Summer**

**10.** $\left(1+\frac{x}{8}\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(\frac{x}{8}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{1.2}\left(\frac{x}{8}\right)^2+\ldots$

$= 1+\frac{x}{16}-\frac{x^2}{512}+\ldots$ B1 $\left(1+\frac{x}{16}\right)$

B1 $\left(-\frac{x^2}{512}\right)$

Expansion is valid for $|x|<8$ B1 (only for conditions on $|x|$)

$\left(1+\frac{1}{8}\right)^{\frac{1}{2}}=1+\frac{1}{6}-\frac{1}{512}$

$\frac{3}{2\sqrt{2}}=\frac{543}{512}$ (o.e.) B1 (expression must involve $\sqrt{2}$)

$\sqrt{2}=\frac{3}{2}\times\frac{512}{543}=\frac{256}{181}$ B1 (convincing)

**5**

**2007 Summer**

**4.** $(1+4x)^{\frac{1}{2}}-\frac{1}{1+3x}$

$= 1+\left(\frac{1}{2}\right)(4x)+\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\frac{1}{2!}(4x)^2+\ldots$ B2 (–1) each error

$-(1-3x+\frac{(-1)(-2)}{2}(3x)^2+\ldots)$ B2 (–1) each error

(correct expansion of $(1+3x)^{-n}$)

$= 1+2x-2x^2+\ldots$

$-1+3x-9x^2+\ldots$ B2

$= 5x-11x^2+\ldots$ (–1 each error)

Expansion valid for $|x|<\frac{1}{4}$ (o.e.) B1

**7**

**2008 Summer**

**9.** $(1+3x)(1-2x)^{-\frac{1}{2}} = (1+3x)\left(1+\left(-\frac{1}{2}\right)(-2x)+\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{1}{2!}1(-2x)^2+\cdots\right)$ B1, B1 (unsimplified)

$= (1+3x)\left(1+x+\frac{3}{2}x^2+\cdots\right)$

$= 1+4x+\frac{9x^2}{2}+\cdots$ B1 $(1+4x)$

B1 $(\frac{9x^2}{2})$

Exparsim is valid for $|x|<\frac{1}{2}$ B1

**5**

**2009 Summer**

**9.** $(1+4x)^{\frac{1}{2}} = 1+\frac{1}{2}(4x)+\frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2}(4x)^2+\ldots$ (first line with possibly $4x^2$) M1

$= 1+2x-2x^2+\ldots.$ $(1+2x)$ A1

$(-2x^2)$ A1

Valid for $|x|<\frac{1}{4}$ B1

$(1+4k+16k^2) = 1+2(k+4k^2)-2(k+4k^2)^2+\ldots$ (correct substitution for $x$ and attempt to evaluate) M1

$= 1+2k+8k^2-2k^2+\ldots.$

$= 1+2k+6k^2+\ldots.$ (F.T. quadratic in $x$) A1

[ **Alternative:**
First principles with three terms M1
Answer A1 ]

**2010 Summer**

**5.** $\left(1-\frac{x}{4}\right)^{1/2} = 1-\frac{x}{8}-\frac{x^2}{128}$ $\left(1-\frac{x}{8}\right)$ B1

$\left(-\frac{x^2}{128}\right)$ B1

$|x| \leq 4$ or $-4 < x < 4$ B1

$\frac{\sqrt{3}}{2} \approx 1-\frac{1}{8}-\frac{1}{128}$ (f.t. candidate's coefficients) B1

$\sqrt{3} \approx \frac{111}{64}$. (convincing) B1

**2011 Summer**

**6.** $(1+2x)^{1/2} = 1+(1/2)\times(2x)+\frac{(1/2)\times(1/2-1)\times(2x)^2}{1\times 2}+\ldots$

(– 1 each incorrect term) B2

$\frac{1}{(1+3x)^2} = 1+(-2)\times(3x)+\frac{(-2)\times(-3)\times(3x)^2}{1\times 2}+\ldots$

(– 1 each incorrect term) B2

$4(1+2x)^{1/2}-\frac{1}{(1+3x)^2} = 3+10x-29x^2+\ldots$

(– 1 each incorrect term) B2

Expansion valid for $|x| < 1/3$ B1

**2012 Summer**

**5.** $\left(1+\frac{x}{3}\right)^{-1/2} = 1 - \frac{x}{6} + \frac{x^2}{24}$ $\left(1-\frac{x}{6}\right)$ B1

$\left(\frac{x^2}{24}\right)$ B1

$|x| < 3$ or $-3 < x < 3$ B1

$\left(\frac{16}{15}\right)^{-1/2} \approx 1 - \frac{1}{30} + \frac{1}{600}$ (f.t. candidate's coefficients) B1

$\sqrt{15} \approx \frac{581}{150}$ (c.a.o.) B1

**2013 Summer**

**5.** (*a*) (i) $(1+6x)^{1/3} = 1 + 2x - 4x^2$ $(1+2x)$ B1

$(-4x^2)$ B1

(ii) $|x| < {}^1/_6$ or $-{}^1/_6 < x < {}^1/_6$ B1

(*b*) $2 + 4x - 8x^2 = 2x^2 - 15x \Rightarrow 10x^2 - 19x - 2 = 0$ M1

(An attempt to set up and use a correct method to solve quadratic using candidate's expansion for $(1+6x)^{1/3}$)

$x = -0{\cdot}1$ (f.t. only candidate's range for $x$ in (*a*)) A1

**2014 Summer**

**5.** $(1-2x)^{1/2} = 1 + (1/2) \times (-2x) + \frac{(1/2) \times (1/2 - 1) \times (-2x)^2}{1 \times 2} + \ldots$

(–1 each incorrect term) B2

$\frac{1}{1+4x} = 1 + (-1) \times (4x) + \frac{(-1) \times (-2) \times (4x)^2}{1 \times 2} + \ldots$

(–1 each incorrect term) B2

$6\sqrt{1-2x} - \frac{1}{1+4x} = 5 - 2x - 19x^2 + \ldots$ (–1 each incorrect term) B2

Expansion valid for $|x| < 1/4$ (o.e.) B1

**2015**

**5.** $\left(1+\frac{x}{8}\right)^{-1/2} = 1 - \frac{x}{16} + \frac{3x^2}{512}$ $\left(1-\frac{x}{16}\right)$ B1

$\left(\frac{3x^2}{512}\right)$ B1

$|x| < 8$ or $-8 < x < 8$ B1

$\frac{2}{3}\sqrt{2} \approx 1 - \frac{1}{16} + \frac{3}{512}$ (f.t. candidate's coefficients) B1

**Either:** $\sqrt{2} \approx \frac{1449}{1024}$ (c.a.o.)

**Or:** $\sqrt{2} \approx \frac{2048}{1449}$ (c.a.o.) B1

**2016**

**2.** (*a*) (i) $(1+2x)^{-1/2} = 1 - x + \frac{3x^2}{2}$ $(1-x)$ B1

$\left(\frac{3}{2}x^2\right)$ B1

(ii) $|x| < \frac{1}{2}$ or $-\frac{1}{2} < x < \frac{1}{2}$ B1

(*b*) $6 - 6x + 9x^2 = 4 + 15x - x^2 \Rightarrow 10x^2 - 21x + 2 = 0$

(f.t. only candidate's quadratic expansion in (*a*)) M1

$x = 0{\cdot}1$ (f.t. only candidate's quadratic expansion in (*a*)) A1

**2017**

**5.** (*a*) $(1+4x)^{-1/2} = 1 - 2x + 6x^2 + \ldots$ $(1-2x)$ B1

$(6x^2)$ B1

$|x| < \frac{1}{4}$ or $-\frac{1}{4} < x < \frac{1}{4}$ B1

(*b*) $1 + 4y + 8y^2 = 1 + 4(y + 2y^2)$ M1

$(1 + 4y + 8y^2)^{-1/2} = 1 - 2(y + 2y^2) + 6(y + 2y^2)^2 + \ldots$

(f.t. candidate's expression from part (*a*)) m1

$(1 + 4y + 8y^2)^{-1/2} = 1 - 2y + 2y^2 + \ldots$

(f.t. candidate's expression from part (*a*)) A1