C4 Differential Equations Answers

Specimen

6. *(a)* $\frac{dx}{dt} = -kx$ B1

Now at $t = 0$, $x = 2$, $\frac{dx}{dt} = -0 \cdot 064$ M1 (use of data)

Then $k = \frac{-0.064}{-2} = 0 \cdot 032$

$\therefore \frac{dx}{dt} = -0 \cdot 032x$ A1

(b) $\int \frac{dx}{x} = \int -0 \cdot 032 dt$ M1 (separation of variables and integration of r.h.s.)

$\therefore \ln x = -0 \cdot 032t + A$ A1

$t = 0, x = 2$

$\therefore A = \ln 2$ M1

$\therefore \ln x = -0 \cdot 032t + \ln 2$

$\therefore 0.032t = \ln 2 - \ln x$

$= \ln \frac{2}{x}$ A1

$\therefore t = \frac{1}{0 \cdot 032} \ln \frac{2}{x}$

$= \frac{125}{4} \ln \frac{2}{x}$ A1

(c) $x = 1, t = \frac{125}{4} \ln 2$ M1

$\approx 21 \cdot 66$ years A1

2005 Summer

8.	(a)	$\frac{dP}{dt} = kP$	B1
	(b)	$\int \frac{dP}{P} = \int k dt$	M1
		$\ln P = kt \ (+C)$	A1
		$t = 0, P = P_0 \quad \ln P_0 = C$	M1
		$\ln P = kt + \ln P_0$	
		$\ln\left(\frac{P}{P_0}\right) = kt$	A1
		$\frac{P}{P_0} = e^{kt}$	
		$P = P_0 e^{kt}$	A1 (convincing)
	(c)	$1.2 P_0 = P_0 e^{2k}$	M1 (attempt to find k)
		$2k = \ln 1.2$	
		$k = \frac{1}{2}\ln 1.2$	A1
		$T = \frac{\ln 2}{\frac{1}{2}\ln 1.2} \approx 7.6$	m1 A1 (F.T. one slip)
			[10]

2006 Summer

8.	*(a)*	$\frac{dx}{dt} = -k\sqrt{x}$	B1
	(b)	$\int \frac{dx}{\sqrt{x}} = \int -k dt$	M1 (attempt to separate variables, allow similar work)
		$2x^{\frac{1}{2}} = -kt + C$	A1 (unsimplified version, allow absence of C)
		$t = 0, \ x = 9, \quad 2\sqrt{9} = C$	M1 (correct attempt to find C)
		$C = 6$	
		$kt = 6 - 2\sqrt{x}$	A1 (convincing)

(c)	$t = 20,\ x = 4$ gives $k = \dfrac{1}{10}$ (o.e.)	M1 (attempt to find k)
		A1
	Tank is empty when $6 = \dfrac{1}{10}t$, $t = 60$ (mins.)	A1
		8

2007 Summer

8.	*(a)*	$\dfrac{dP}{dt} = kP$	B1
	(b)	$\displaystyle\int \frac{dP}{P} = \int k\,dt$	M1 (Separation of variables and ∫)
		$\ln P = kt + C$	A1 F.T. right hand side)
		$t = 0,\ P = 50$	
		$\ln 50 = C$	M1 (attempt to find C)
		$\ln P - \ln 50 = kt$	
		$\ln \dfrac{P}{50} = kt$	M1 (combination of logs and
		$\dfrac{P}{50} = e^{kt}$	attempt to exponentiate)
		$P = 50e^{kt}$	A1
	(c)	$65 = 50\,e^{7k}$	M1 (taking logs correctly)
		$\ln \dfrac{65}{50} = 7k$ $\quad k = 0.03748$	A1
		After sixteen years, $P = 50 \exp(\cdot 03748 \times 16)$	M1 (use of formula)
		$\approx$ £91 (nearest pound)	A1 (C.A.O.)
			10

2008 Summer

7.	*(a)*	$\frac{dW}{dt} = kW \qquad (k > 0)$	B1
	(b)	$\int \frac{dW}{W} = \int k dt$	M1 (attempt to separate variables)
		$\ln W = kt + C$	A1 (allow absence of *C*)
		$t = 0, \quad W = 0.1, \quad C = \ln 0.1$	B1 (value of *C*)
		$\ln \frac{W}{0.1} = kt$	M1 (use of logs or exponentials)
		$\frac{W}{0.1} = e^{kt}$	
		$k = 3.0007$	B1 (value of k)
		$W = 0.1e^{3t}$	A1
			7

2009 Summer

7.	*(a)*	$\frac{dP}{dt} = -kP^3$	(allow $\pm k$)	B1
	(b)	$\int \frac{dP}{p^3} = -\int k\, dt$	(separation of variables & attempt to integrate $\frac{1}{p^n}$, any n)	M1
		$-\frac{1}{2p^2} = -kt + C$	(C may be omitted, $n \neq 1$)	A1
		$t = 0,\ P = 20$	(attempt to find C)	M1
		$\therefore\ -\frac{1}{800} = C$	(F.T. similar work)	A1
		$\therefore\ -\frac{1}{2p^2} = -kt - \frac{1}{800}$		
		$\therefore\ \frac{1}{p^2} = 2kt + \frac{1}{400}$		
		$\frac{1}{p^2} = At + \frac{1}{400} \qquad (A = 2k)$	(convincing)	A1

(c) $t = 1,\ P = 10$

$\frac{1}{100} = A + \frac{1}{400}$ (attempt to find A) M1

$\therefore\ A = \frac{3}{400}$ A1

$\frac{1}{25} = \frac{3}{400} + \frac{1}{400}$ (substitute $p = 5$) m1

$\frac{15}{400} = \frac{3}{400}t$

$t = 5$ (F.T. one slip) A1

2010 Summer

8. (a) $\frac{dV}{dt} = -kV^2$ B1

(b) $\int \frac{dV}{V^2} = -\int k\,dt$ (o.e.) M1

$-\frac{1}{V} = -kt + c$ A1

$c = -\frac{1}{12000}$ (c.a.o.) A1

$V = \frac{12000}{12000kt + 1} = \frac{12000}{at + 1}$ (convincing) A1

(c) Substituting $t = 2$ and $V = 9000$ in expression for V M1

$a = \frac{1}{6}$ A1

Substituting $t = 4$ in expression for V with candidate's value for a M1

$V = 7200$ (c.a.o) A1

2011 Summer

8. (a) $\frac{dN}{dt} = kN$ B1

(b) $\int \frac{dN}{N} = \int k\,dt$ M1

$\ln N = kt + c$ A1

$N = e^{kt+c} = Ae^{kt}$ (convincing) A1 (*c*) (i) $100 = Ae^{2k}$

$160 = Ae^{12k}$ (both values) B1

Dividing to eliminate A M1

$1{\cdot}6 = e^{10k}$ A1

$k = \frac{1}{10}\ln 1{\cdot}6 = 0{\cdot}047$ (convincing) A1

(ii) $A = 91({\cdot}0283)$ (o.e.) B1

When $t = 20$, $N = 91({\cdot}0283) \times e^{0{\cdot}94}$

(f.t. candidate's derived value for A) M1

$N = 233$ (c.a.o.) A1

2012 Summer

8. (*a*) $\frac{dV}{dt} = -kV^3$ (where $k > 0$) B1

(*b*) $\int \frac{dV}{V^3} = -\int k\,dt$ (o.e.) M1

$-\frac{V^{-2}}{2} = -kt + c$ A1

$c = -\frac{1}{7200}$ (c.a.o.) A1

$V^2 = \frac{3600}{7200kt + 1} = \frac{3600}{at + 1}$ (convincing)

where $a = 7200k$ A1

(*c*) Substituting $t = 2$ and $V = 50$ in expression for V^2 M1

$a = 0{\cdot}22$ A1

Substituting $V = 27$ in expression for V^2 with candidate's value for a

M1

$t = 17{\cdot}9$ (c.a.o) A1

2013 Summer

8. (*a*) $\frac{dA}{dt} = k\sqrt{A}$ B1

(*b*) $\int \frac{dA}{\sqrt{A}} = \int k\,dt$ M1

$\frac{A^{1/2}}{1/2} = kt + c$ A1

Substituting 64 for A and 3 for t and 196 for A and 5·5 for t in candidate's derived equation m1

$16 = 3k + c$, $28 = 5{\cdot}5k + c$ (both equations) (c.a.o.) A1

Attempting to solve candidate's derived simultaneous linear equations in k and c

m1

$A = (2{\cdot}4t + 0{\cdot}8)^2$ (o.e.) (c.a.o.) A1

2014 Summer

8. (*a*) $\frac{dV}{dt} = kV$ — B1

(*b*) $\int \frac{dV}{V} = \int k \, dt$ — M1

$\ln V = kt + c$ — A1

$V = e^{kt+c} = Ae^{kt}$ (convincing) — A1

(*c*) (i) $292 = Ae^{2k}$

$637 = Ae^{28k}$ (both values) — B1

Dividing to eliminate A — M1

$\frac{637}{292} = e^{26k}$ — A1

$k = \frac{1}{26} \ln \left[\frac{637}{292}\right] = 0{\cdot}03$ — A1

(ii) $A = 275$ — B1

(iii) When $t = 0$, initial value of investment = £275

(f.t. candidate's derived value for A) — B1

2015

(f.t. one slip) A1

9. (*a*) $\frac{dP}{dt} = kP^2$ — B1

(*b*) $\int \frac{dP}{P^2} = \int k \, dt$ — M1

$-\frac{1}{P} = kt + c$ (o.e.) — A1

$c = -\frac{1}{A}$ (c.a.o.) — A1

$-\frac{1}{P} = kt - \frac{1}{A} \Rightarrow kt = \frac{1}{A} - \frac{1}{P} \Rightarrow \frac{1}{k}\left(\frac{P-A}{PA}\right) = t$ (convincing) — A1

(*c*) $\frac{1}{k}\left(\frac{800-A}{800A}\right) = 3$, $\frac{1}{k}\left(\frac{900-A}{900A}\right) = 4$ (both equations) — B1

An attempt to solve these equations simultaneously by eliminating k — M1

$A = 600$ (c.a.o.) — A1

2016

7. (*a*) $\frac{dV}{dt} = -kV^3$ B1

(*b*) $\int \frac{dV}{V^3} = -\int k\,dt$ (o.e.) M1

$-\frac{V^{-2}}{2} = -kt + c$ A1

$c = -\frac{A^{-2}}{2}$ (c.a.o.) A1

$2V^2 = \frac{2A^2}{(2A^2k)t + 1} \Rightarrow V^2 = \frac{A^2}{bt + 1}$ (convincing)

where $b = 2A^2k$ A1

(*c*) Substituting $t = 2$ and $V = \frac{A}{2}$ in an expression for V^2 M1

$b = \frac{3}{2}$ $\left\{\text{or } k = \frac{3}{4A^2}\right\}$ A1

Substituting $V = \frac{A}{4}$ in an expression for V^2 with candidate's value for b or expression for k M1

$t = 10$ (c.a.o) A1

2017

8. (*a*) $\frac{dN}{dt} = k\sqrt{N}$ B1

(*b*) $\int \frac{dN}{\sqrt{N}} = \int k\,dt$ M1

$\frac{N^{1/2}}{1/2} = kt + c$ A1

Substituting 256 for N and 5 for t and 400 for N and 7 for t in candidate's derived equation m1

$32 = 5k + c$, $40 = 7k + c$ (both equations) (c.a.o.) A1

Attempting to solve candidate's derived simultaneous linear equations in k and c $(k = 4, c = 12)$ m1

$N = (2t + 6)^2$ (o.e.) (c.a.o.) A1