

C4 Differential Equations Questions

Specimen

6. Actinium is a radioactive substance which decays slowly.

Initially, 2 kg of actinium is present and the rate of decay of its mass is 64 g/year. Subsequently, t years later when the actinium has a mass x kg, the rate of decrease of mass is proportional to the value of x .

(a) Show that $\frac{dx}{dt} = -0.032x$. [3]

(b) Deduce that $t = \frac{125}{4} \ln\left(\frac{2}{x}\right)$. [5]

- (c) Find the value of t when half the actinium has decayed, giving your answer correct to two decimal places. [2]

2005 Summer

8. The size P of a population of bacteria at time t days is to be modelled as a continuous variable such that the rate of increase of P is directly proportional to P .

(a) Write down a differential equation that is satisfied by P . [1]

(b) Given that the initial size of the population is P_0 , show that $P = P_0 e^{kt}$, where k is a positive constant. [5]

(c) Two days after the start, the population is $1.2P_0$. Find when the population will be $2P_0$. [4]

2006 Summer

8. Water leaks from a hole at the bottom of a large water tank. The depth of the water at time t minutes is x metres. The rate of decrease of x is directly proportional to \sqrt{x} .

(a) Write down a differential equation that is satisfied by x . [1]

- (b) Given that the depth of water in the tank when $t = 0$ is 9 metres, show that

$$kt = 6 - 2\sqrt{x},$$

where k is a positive constant. [4]

- (c) Given that the depth of water in the tank is 4 metres when $t = 20$, find the time taken for the tank to empty. [3]

2007 Summer

8. The price $\pounds P$ of an item at time t years is to be modelled as a continuous variable such that the rate of increase of P is directly proportional to P .
- (a) Write down a differential equation that is satisfied by P . [1]
- (b) Given that the price of the item at $t = 0$ is $\pounds 50$, show that $P = 50e^{kt}$, where k is a positive constant. [5]
- (c) After seven years the price of the item is $\pounds 65$. Find the price of the item after sixteen years. [4]

2008 Summer

7. A neglected large lawn contains a certain type of weed. The area of the lawn covered by the weed at time t years is $W \text{ m}^2$. The rate of increase of W is directly proportional to W .
- (a) Write down a differential equation that is satisfied by W . [1]
- (b) The area of the lawn covered by the weed initially is 0.10 m^2 and one year later the area covered is 2.01 m^2 . Find an expression for W in terms of t . [6]

2009 Summer

7. The value of an electronic component may be modelled as a continuous variable. The value of the component at time t years is $\pounds P$. The rate of decrease of P is directly proportional to P^3 .
- (a) Write down a differential equation that is satisfied by P . [1]
- (b) The value of the component when $t = 0$ is $\pounds 20$. Show that
$$\frac{1}{P^2} = \frac{1}{400} + At,$$
 where A is a positive constant. [5]
- (c) Given that the value of the component when $t = 1$ is $\pounds 10$, find the time when the value is $\pounds 5$. [4]

2010 Summer

8. The value, £ V , of a car may be modelled as a continuous variable. At time t years, the rate of decrease of V is directly proportional to V^2 .

(a) Write down a differential equation satisfied by V . [1]

(b) Given that $V = 12000$ when $t = 0$, show that

$$V = \frac{12000}{at + 1},$$

where a is a constant. [4]

(c) The value of the car at the end of two years is £9000. Find the value of the car at the end of four years. [4]

2011 Summer

8. The size N of the population of a small island may be modelled as a continuous variable. At time t , the rate of increase of N is directly proportional to the value of N .

(a) Write down the differential equation that is satisfied by N . [1]

(b) Show that $N = Ae^{kt}$, where A and k are constants. [3]

(c) Given that $N = 100$ when $t = 2$ and that $N = 160$ when $t = 12$,
(i) show that $k = 0.047$, correct to three decimal places,
(ii) find the size of the population when $t = 20$. [7]

2012 Summer

8. Water is leaking from a hole at the bottom of a large tank. The volume of the water in the tank at time t hours is $V\text{m}^3$. The rate of decrease of V is directly proportional to V^3 .

(a) Write down a differential equation satisfied by V . [1]

(b) Given that $V = 60$ when $t = 0$, show that
$$V^2 = \frac{3600}{at + 1},$$
where a is a constant. [4]

(c) When $t = 2$, the volume of the water in the tank is 50m^3 . Find the value of t when the volume of the water in the tank is 27m^3 . Give your answer correct to one decimal place. [4]

2013 Summer

8. Part of the surface of a small lake is covered by green algae. The area of the lake covered by the algae at time t years is $A \text{ m}^2$. The rate of increase of A is directly proportional to \sqrt{A} .
- (a) Write down a differential equation satisfied by A . [1]
- (b) The area of the lake covered by the algae at time $t = 3$ is 64 m^2 and the area covered at time $t = 5.5$ is 196 m^2 . Find an expression for A in terms of t . [6]

2014 Summer

8. The value $\text{£}V$ of a long term investment may be modelled as a continuous variable. At time t years, the rate of increase of V is directly proportional to the value of V .
- (a) Write down a differential equation satisfied by V . [1]
- (b) Show that $V = Ae^{kt}$, where A and k are constants. [3]
- (c) The value of the investment after 2 years is $\text{£}292$ and its value after 28 years is $\text{£}637$.
- (i) Show that $k = 0.03$, correct to two decimal places.
- (ii) Find the value of A correct to the nearest integer.
- (iii) Find the initial value of the investment. Give your answer correct to the nearest pound. [6]

2015

9. A bookseller values a rare book at $\text{£}A$ on August 1st 2010. The value, $\text{£}P$, of the book t years after this date may be modelled as a continuous variable. The rate of increase of P may be assumed to be directly proportional to P^2 .
- (a) Write down a differential equation satisfied by P . [1]
- (b) Show that
- $$\frac{1}{k} \left(\frac{P-A}{PA} \right) = t,$$
- where k is a constant. [4]
- (c) The value of the book is $\text{£}800$ on August 1st 2013 and $\text{£}900$ on August 1st 2014. Find the value of A . [3]

2016

7. The value, £ V , of a particular car may be modelled as a continuous variable. At time t years, the rate of decrease of V is directly proportional to V^3 .

(a) Write down a differential equation satisfied by V . [1]

(b) Given that the initial value of the car is £ A , show that

$$V^2 = \frac{A^2}{bt+1},$$

where b is a constant. [4]

(c) When $t = 2$, the value of the car has fallen to a half of its initial value. Find the value of t when the value of the car will have fallen to a quarter of its initial value. [4]

2017

8. The size N of the population of a small island may be modelled as a continuous variable. At time t years, the rate of increase of N is assumed to be directly proportional to the value of \sqrt{N} .

(a) Write down a differential equation satisfied by N . [1]

(b) When $t = 5$, the size of the population was 256. When $t = 7$, the size of the population was 400. Find an expression for N in terms of t . [6]