

C4 Integration Questions

Specimen

8. (a) Show that

$$\int_0^{\frac{\pi}{4}} \cos^2 \theta \, d\theta = \frac{\pi}{8} + \frac{1}{4}. \quad [4]$$

- (b) Use the substitution $x = 3\tan\theta$ to evaluate

$$\int_0^3 \frac{27}{(9+x^2)^2} dx. \quad [6]$$

2005 Summer

7. (a) Use the substitution $u = 2x - 1$ to evaluate

$$\int_0^1 x(2x-1)^9 dx. \quad [5]$$

- (b) (i) Find $\int x \cos 2x \, dx$. [4]

- (ii) Use the result of (b)(i) to find $\int x \cos^2 x \, dx$. [3]

2006 Summer

7. (a) Find $\int x \ln x \, dx$. [5]

- (b) Use the substitution $u = 2\sin x + 3$ to evaluate

$$\int_0^{\frac{\pi}{6}} \frac{\cos x}{(2\sin x + 3)^2} dx. \quad [4]$$

2007 Summer

7. (a) Find $\int x^2 \ln x \, dx$. [4]

- (b) Use the substitution $x = 2\sin\theta$ to show that

$$\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx = \int_0^a k \sin^2 \theta d\theta,$$

where the values of a and k are to be determined.

- Hence, or otherwise, evaluate $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx$. [8]

2008 Summer

6. (a) Find $\int (3x+1)e^{2x} dx$. [4]

(b) Use the substitution $x = 3\sin\theta$ to show that

$$\int_{1.5}^3 \sqrt{9-x^2} dx = \int_a^b k \cos^2 \theta d\theta$$

where the values of the constants a , b and k are to be found.

Hence evaluate $\int_{1.5}^3 \sqrt{9-x^2} dx$. [8]

2009 Summer

6. (a) Find $\int (x+3)e^{2x} dx$. [4]

(b) Use the substitution $u = 2\cos x + 1$ to evaluate

$$\int_0^{\frac{\pi}{3}} \frac{\sin x}{\sqrt{(2\cos x + 1)}} dx$$
 [5]

2010 Summer

7. (a) Find $\int x^3 \ln x dx$. [4]

(b) Use the substitution $u = 2x - 3$ to evaluate $\int_1^2 x(2x-3)^4 dx$. [5]

2011 Summer

7. (a) Find $\int x \sin 2x dx$. [4]

(b) Use the substitution $u = 5 - x^2$ to evaluate

$$\int_0^2 \frac{x}{(5-x^2)^3} dx$$
 [4]

2012 Summer

7. (a) Find $\int xe^{-2x} dx$. [4]

(b) Use the substitution $u = 1 + 3 \ln x$ to evaluate

$$\int_1^e \frac{1}{x(1+3 \ln x)} dx.$$

Give your answer correct to four decimal places. [4]

2013 Summer

7. (a) Find $\int (3x - 1) \cos 2x dx$. [4]

(b) Use the substitution $u = 2x + 1$ to evaluate

$$\int_0^1 \frac{x}{(2x+1)^3} dx. [5]$$

2014 Summer

7. (a) Find $\int x^4 \ln 2x dx$. [4]

(b) Use the substitution $u = 10 \cos x - 1$ to evaluate

$$\int_0^{\frac{\pi}{3}} \sqrt{(10 \cos x - 1)} \sin x dx. [4]$$

2015

7. (a) Use the substitution $u = 12 - x^3$ to evaluate

$$\int_0^2 \frac{x^2}{(12 - x^3)^2} dx. [4]$$

(b) (i) Find $\int x \cos 2x dx$.

(ii) Use the result of (b)(i) to find

$$\int x \sin^2 x dx. [7]$$

2016

6. (a) Find $\int (2x+1)e^{-3x} dx$. [4]

(b) Use the substitution $u = 4 + 5 \tan x$ to evaluate

$$\int_0^{\frac{\pi}{4}} \frac{\sqrt{4+5 \tan x}}{\cos^2 x} dx. \quad [4]$$

2017

7. (a) Find $\int \frac{\ln x}{x^4} dx$. [4]

(b) Use the substitution $u = x^2 + 1$ to evaluate

$$\int_0^1 x^3 (x^2 + 1)^4 dx. \quad [5]$$

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