C4 Normal and Tangents Answers

2005

3. $8x + 3y + 3x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	B1 $(2y\frac{dy}{dx})$
	B1 $(3y + 3x\frac{dy}{dx})$
$\frac{dy}{dx} = \frac{8x+3y}{2y-3x}$	
$= -\frac{19}{4}$	B1 (C.A.O.)
Equation is $y - 1 = -\frac{19}{4}(x-2)$	B1 (F.T one slip)
	[4]

2006

2. $6x^2 + 6y^2 + 12xy\frac{dy}{dx} - 4y^2\frac{dy}{dx} = 0$	B1 $(4y^3\frac{dy}{dx})$
	B1 $(6y^2 + 12xy\frac{dy}{dx})$
$\frac{dy}{dx} = -\frac{3}{2}$	B1 (C.A.O.)
Gradient of normal $= \frac{3}{2}$	M1 (F.T. candidate's $\frac{dy}{dx}$)
Equation is $y - 1 = \frac{2}{3}(x-2)$	A1 (F.T. candidate's gradient of normal)

2007

2. $5x^4 + y^2 + 2xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$		B1 $(y^2 + 2xy\frac{dy}{dx})$
		B1 $(3y^2\frac{dy}{dx})$
$\frac{dy}{dx} = -\frac{2}{3}$	(o.e.)	B1 (C.A.O.)
Equation is $y - 3 = -\frac{2}{3}(x+1)$		B1 (F.T. candidate's $\frac{dy}{dx}$)

2008

2. $2x + x\frac{dy}{dx} + y + 4y\frac{dy}{dx} = 0$ — B1 $(x\frac{dy}{dx} + y)$

B1 $(4y\frac{dy}{dx})$

$\frac{dy}{dx} = 5$ — B1 (C.A.O.)

Gradient of normal $= -\frac{1}{5}$ — M1 $\left(\frac{-1}{candidate's\ \frac{dy}{dx}}\right.$, *numerical value*)

Equation of normal is $y - 1 = -\frac{1}{5}(x + 3)$ — A1 (F.T. candidate's value)

2010

2. $10x + 4x\frac{dy}{dx} + 4y - 3y^2\frac{dy}{dx} = 0$ — $\left[4x\frac{dy}{dx} + 4y\right]$ B1

$\left[-3y^2\frac{dy}{dx}\right]$ B1

$\frac{dy}{dx} = \frac{1}{4}$ (o.e.) (c.a.o.) B1

Use of $grad_{normal} \times grad_{tangent} = -1$ — M1

Equation of normal: $y - (-2) = -4(x - 1)$

$\left[\text{f.t. candidate's value for } \frac{dy}{dx}\right]$ A1

2011

2. $4x^3 - 2x^2\frac{dy}{dx} - 4xy + 2y\frac{dy}{dx} = 0$ — $\left[-2x^2\frac{dy}{dx} - 4xy\right]$ B1

$\left[4x^3 + 2y\frac{dy}{dx}\right]$ B1

Either $\frac{dy}{dx} = \frac{4xy - 4x^3}{2y - 2x^2}$ **or** $\frac{dy}{dx} = 2$ (o.e.) (c.a.o.) B1

Attempting to substitute $x = 1$ and $y = 3$ in candidate's expression **and** the use of $grad_{normal} \times grad_{tangent} = -1$ — M1

Equation of normal: $y - 3 = -\frac{1}{2}(x - 1)$

$\left[\text{f.t. candidate's value for } \frac{dy}{dx}\right]$ A1

2012

2. $3y^2\frac{dy}{dx} - 8x - 3x\frac{dy}{dx} - 3y = 0$

$\left(3y^2\frac{dy}{dx} - 8x\right)$ B1

$\left(-3x\frac{dy}{dx} - 3y\right)$ B1

Either $\frac{dy}{dx} = \frac{3y + 8x}{3y^2 - 3x}$ **or** $\frac{dy}{dx} = \frac{1}{3}$ (o.e.) (c.a.o.) B1

Equation of tangent: $y - (-3) = \frac{1}{3}(x - 2)$

$\left(\text{f.t. candidate's value for } \frac{dy}{dx}\right)$ B1

2013

2. $3x^2 - 2x \times 2y\frac{dy}{dx} - 2y^2 + 3y^2\frac{dy}{dx} = 0$

$\left(-2x \times 2y\frac{dy}{dx} - 2y^2\right)$ B1

$\left(3x^2, 3y^2\frac{dy}{dx}\right)$ B1

Either $\frac{dy}{dx} = \frac{2y^2 - 3x^2}{3y^2 - 4xy}$ **or** $\frac{dy}{dx} = 2$ (o.e.) (c.a.o.) B1

Use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ M1

Equation of normal: $y - 1 = -\frac{1}{2}(x - 2)$ $\left(\text{f.t. candidate's value for } \frac{dy}{dx}\right)$ A1

2014

1. $9x^2 - 5x \times 2y\frac{dy}{dx} - 5y^2 + 8y^3\frac{dy}{dx} = 0$

$\left(-5x \times 2y\frac{dy}{dx} - 5y^2\right)$ B1

$\left(9x^2 + 8y^3\frac{dy}{dx}\right)$ B1

Either $\frac{dy}{dx} = \frac{9x^2 - 5y^2}{10xy - 8y^3}$ **or** $\frac{dy}{dx} = \frac{1}{4}$ (o.e.) (c.a.o.) B1

Attempting to substitute $x = 1$ and $y = 2$ in candidate's expression **and** the use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ M1

Equation of normal: $y - 2 = -4(x - 1)$

$\left(\text{f.t. candidate's value for } \frac{dy}{dx}\right)$ A1

2015

2. (*a*) $4x^3 + 3x^2\frac{dy}{dx} + 6xy - 4y\frac{dy}{dx} = 0$ $\left[3x^2\frac{dy}{dx} + 6xy\right]$ B1

$\left[4x^3 - 4y\frac{dy}{dx}\right]$ B1

$\frac{dy}{dx} = \frac{4x^3 + 6xy}{4y - 3x^2}$ (convincing) B1

(*b*) $4y - 3x^2 = 0$ M1

Either: Substituting $\frac{3x^2}{4}$ for y in the equation of C and an attempt to collect terms m1

$x^4 = 16 \Rightarrow x = (\pm)\,2$ A1

$y = 3$ (for both values of x)

(f.t. $x^4 = a$, $a \neq 16$, provided both x values are checked) A1

Or: Substituting $\frac{4y}{3}$ for x^2 in the equation of C and an attempt to collect terms m1

$y^2 = 9 \Rightarrow y = (\pm)\,3$ A1

$y = 3 \Rightarrow x = \pm 2$ (f.t. $y^2 = b$, $b \neq 9$) A1

2016

3. (*a*) $4x^3 + 2x^3\frac{dy}{dx} + 6x^2y - 12y^3\frac{dy}{dx} = 0$ $\left[2x^3\frac{dy}{dx} + 6x^2y\right]$ B1

$\left[4x^3 - 12y^3\frac{dy}{dx}\right]$ B1

$\frac{dy}{dx} = \frac{2x^3 + 3x^2y}{6y^3 - x^3}$

(intermediary line required in order to be convincing) B1

(*b*) $2x^3 + 3x^2y = -2(6y^3 - x^3)$ M1

$y(3x^2 + 12y^2) = 0$ A1

$3x^2 + 12y^2 = 0 \Rightarrow x = 0, y = 0$ but not on curve A1

$y = 0 \Rightarrow x = \pm 2 \Rightarrow (2, 0), (-2, 0)$ (both points) A1

2017

2. (*a*) $6y^5\frac{dy}{dx} - 12x^3 - 9x^2\frac{dy}{dx} - 18xy = 0$

$\left[6y^5\frac{dy}{dx} - 12x^3\right]$ B1

$\left[-9x^2\frac{dy}{dx} - 18xy\right]$ B1

$\frac{dy}{dx} = \frac{6xy + 4x^3}{2y^5 - 3x^2}$

(convincing i.e intermediary line required) B1

(*b*) $y = 0 \Rightarrow x = 2$ or $x = -2$ B1

At (2, 0), $\frac{dy}{dx} = -\frac{8}{3}$ B1

At (–2, 0), $\frac{dy}{dx} = \frac{8}{3}$ B1