C4 Parametric Differentiation Answers

Specimen

5.	$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$	M1 $\left(\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}\right)$ A1
	For normal at P, gradient $= -p$	B1
	$\therefore y - 2ap = -p\,(x - ap^2)$	M1
	giving $px + y - 2ap - ap^3 = 0$	A1 (convincing)
	Q: $y = 0, x = 2a + ap^2$	B1
	R: $y = 0, x = ap^2$	B1
	$\therefore QR = 2a$	B1

2005 Summer

6.	(a)	$\frac{dy}{dx} = \frac{2p}{2} = p$	M1 ($\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$, one correct) A1
		Equation is	
		$y - (p^2 + 3) = p\,(x - 2p - 1)$	M1
		$px - y = p^2 + p - 3$	A1 (convincing)
	(b)	$2p + 3 = p^2 + p - 3$	M1
		$p^2 - p - 6 = 0$	A1
		$p = 3, -2$	A1 (FT one slip)
		Choose $p = -2$ (2nd quadrant)	
		Tangent is $2x + y = 1$	A1 (FT candidate's values)
			[8]

2006 Summer

6.	(a)	$\frac{dy}{dx} = \frac{2t}{-\frac{1}{t^2}} - 2t^3$	M1 $\left(\frac{\dot{y}}{\dot{x}}\right)$
			A1 (simplified)
		Equation of tangent is	
		$y - p^2 = -2p^3\left(x - \frac{1}{p}\right)$	M1 $(y - y_1 = m(x - x_1))$ o.e.
		$y - p^2 = -2p^3x + 2p^2$	
		$2p^3x + y - 3p^2 = 0$	A1 (convincing)
	(b)	$y = 0,\ x = \frac{3}{2p}$ (o.e.)	B1
		$x = 0,\ y = 3p^2$	B1
		$PA^2 = \left(\frac{3}{2p} - \frac{1}{p}\right)^2 + (p^2 - 0)^2 = \frac{1}{4p^2} + p^4$ (o.e.)	M1 (correct use of distance formula in context)
			A1 (one correct simplified distance C.A.O.)
		$PB^2 = 4PA^2,\ PB = 2PA$	A1 (convincing)
			9

2007 Summer

6.	(a)	$\frac{dy}{dx} = \frac{2t}{2} = t$	M1 (correct attempt to find gradient)
		Gradient of normal $= -\frac{1}{t}$	A1
		Equation is	
		$y - p^2 = -\frac{1}{p}(x - 2p)$	M1 $(y - y_1 = m(x - x_1))$ o.e.
		$py - p^3 = -x + 2p$	
		$x + py = p^3 + 2p$	A1 (convincing)
	(b)	A $y = 0,\ x = p^3 + 2p$	B1 (must be correct for A and B)
		B $x = 0,\ y = p^2 + 2$	B1
		$p^3 + 2p = 2(p^2 + 2)$	M1 (candidate's OA = $k \times$ candidate's OB, $k = \frac{1}{2}$ or 2)
		$p(p^2 + 2) = 2(p^2 + 2)$	
		$p = 2$	A1 (C.A.O.)
			8

2008 Summer

5. *(a)* $\frac{dy}{dx} = \frac{-2\sin 2t}{4\cos t}$ — M1 $\left(\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}\right)$

B1 ($4\cos t$)

M1 ($k\sin 2t, k = -1, \pm 2, -\frac{1}{2}$)

A1 ($k = -2$)

$= \frac{-4\sin t\cos t}{4\cos t} = -\sin t$ — M1 (correct use of formula)

A1 (C.A.O.)

(b) Equation of tangent is $y - \cos 2p = -\sin p(x - 4\sin p)$ — M1 ($y - y_1 = m(x - x_1)$)

$x\sin p + y = \cos 2p + 4\sin^2 p$

$= 1 - 2\sin^2 p + 4\sin^2 p$ — M1 (attempt to use correct formula)

$= 1 + 2\sin^2 p$ — A1

9

2009 Summer

5. *(a)* $\frac{dy}{dx} = \frac{3t^2}{2t}$ — $\left(\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}\right)$ M1

$= \frac{3t}{2}$ — (simplified form) A1

Equation of tangent is

$y - p^3 = \frac{3}{2}p(x - p^2)$ — (use of any method) M1

$2y - 2p^3 = 3px - 3p^3$

$3px - 2y = p^3$ — (convincing) A1

(*b*) Substitute $x = q^2,\ y = q^3$ (substitution of $x = q^2,\ y = q^3$ and $p = 2$) M1

$3pq^2 - 2q^3 = p^3$

When $p = 2$,

$6q^2 - 2q^3 = 8$

$q^3 - 3q^2 + 4 = 0$ (convincing) A1

$(q+1)(q^2 - 4q + 4) = 0$ (attempt to solve) M1

$q = -1$ or $q = 2$ A1

Disregard $q = 2$ (as this relates to point *P*) A1

[Alternatively:

$\frac{y - q^3}{x - q^2} = 3$ (must have gradient 3) M1

$q^3 - 3q^2 + 4 = 0$ (convincing) A1 **]**

2010 Summer

6. (*a*) Use of $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ and at least one of $\frac{dx}{dt} = -\frac{2}{t^2}$, $\frac{dy}{dt} = 4$ correct M1

$\frac{dy}{dx} = -2t^2$ (o.e.) A1

Equation of tangent at *P*: $y - 4p = -2p^2\left(x - \frac{2}{p}\right)$

(f.t. candidate's expression for $\frac{dy}{dx}$) m1

$y = -2p^2x + 8p$ (convincing) A1

(*b*) Substituting $x = 2,\ y = 3$ in equation of tangent M1

$4p^2 - 8p + 3 = 0$ A1

$p = \frac{1}{2}, \frac{3}{2}$ (both values, c.a.o.) A1

Points are $(4, 2)$, $(^4/_3, 6)$ (f.t. candidate's values for *p*) A1

2011 Summer

4. (*a*) candidate's *x*-derivative = – 3 sin *t*
candidate's *y*-derivative = 4 cos *t* (at least one term correct) B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = -\frac{4\cos t}{3\sin t}$ (o.e.) (c.a.o.) A1

At *P*, $y - 4\sin p = -\frac{4\cos p}{3\sin p}(x - 3\cos p)$ (o.e.)

(f.t. candidate's expression for $\frac{dy}{dx}$) M1

$(3\sin p)y - 12\sin^2 p = (-4\cos p)x + 12\cos^2 p$
$(3\sin p)y = (-4\cos p)x + 12\cos^2 p + 12\sin^2 p$
$(3\sin p)y + (4\cos p)x - 12 = 0$ (convincing) A1

(*b*) (i) $A = (2\sqrt{3}, 0)$ B1
$B = (0, 8)$ B1
(ii) Correct use of Pythagoras Theorem to find *AB* M1
$AB = 2\sqrt{19}$ (convincing) A1

2012 Summer

6. (*a*) candidate's *x*-derivative = 2*t*
candidate's *y*-derivative = 2 (at least one term correct)
and use of
$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{1}{t}$ (o.e.) (c.a.o.) A1

Use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ m1

Equation of normal at *P*: $y - 2p = -p(x - p^2)$
(f.t. candidate's expression for $\frac{dy}{dx}$) m1

$y + px = p^3 + 2p$ (convincing) (c.a.o.) A1

(*b*) (i) Substituting *x* = 9, *y* = 6 in equation of normal M1
$p^3 - 7p - 6 = 0$ (convincing) A1
(ii) A correct method for solving $p^3 - 7p - 6 = 0$ M1
$p = -1$ A1
$p = -2$ A1
P is either (1, – 2) or (4, – 4) (c.a.o.) A1

2013 Summer

6. (*a*) candidate's x-derivative $= a$
candidate's y-derivative $= -\frac{b}{t^2}$ (at least one term correct) B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = -\frac{b}{at^2}$ (c.a.o.) A1

Tangent at P: $y - \frac{b}{p} = -\frac{b}{ap^2}(x - ap)$ (o.e.)
(f.t. candidate's expression for $\frac{dy}{dx}$) M1

$ap^2y - abp = -bx + abp$
$ap^2y + bx - 2abp = 0.$ (convincing) A1

(*b*) $y = 0 \Rightarrow x = 2ap$ (o.e.) B1
$x = 0 \Rightarrow y = 2b/p$ (o.e.) B1
Area of triangle $AOB = 2ab$ (c.a.o.) B1

(*c*) $p^2 - 2p + 2 = 0$ $(abp^2 - 2abp + 2ab = 0)$ B1
Attempting **either** to use the formula to solve the candidate's quadratic in p **or** to find the discriminant of the candidate's quadratic **or** to complete the square M1
Either discriminant < 0 ($\Rightarrow$ no real roots) $\Rightarrow$ no such P can exist **or** $(p - 1)^2 + 1 = 0$ $(\Rightarrow (p - 1)^2 = -1)) \Rightarrow$ no such P can exist
(c.a.o.) A1

2014 Summer

6. (*a*) candidate's x-derivative $= 2$
candidate's y-derivative $= 15t^2$ (at least one term correct)
and use of
$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{15t^2}{2}$ (o.e.) (c.a.o.) A1

Equation of tangent at P: $y - 5p^3 = \frac{15}{2}p^2(x - 2p)$
(f.t. candidate's expression for $\frac{dy}{dx}$) m1

$2y = 15p^2x - 20p^3$ (convincing) A1

(*b*) Substituting $p = 1, x = 2q, y = 5q^3$ in equation of tangent M1
$q^3 - 3q + 2 = 0$ (convincing) A1
Putting $f(q) = q^3 - 3q + 2$
Either $f(q) = (q - 1)(q^2 + q - 2)$ **or** $f(q) = (q + 2)(q^2 - 2q + 1)$ M1
Either $f(q) = (q - 1)(q - 1)(q + 2)$ **or** $q = 1, q = -2$ A1
$q = -2$ A1

2015

6. (*a*) (i) candidate's x-derivative = $2at$
candidate's y-derivative = $2a$ (at least one term correct)
and use of
$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1
$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$
Gradient of tangent at $P = \frac{1}{p}$ (c.a.o.) A1

(ii) Equation of tangent at P: $y - 2ap = \frac{1}{p}(x - ap^2)$
(f.t. candidate's expression for $\frac{dy}{dx}$) m1
Equation of tangent at P: $py = x + ap^2$ A1

(*b*) (i) Gradient $PQ = \frac{2ap - 2aq}{ap^2 - aq^2}$ B1
Use of $ap^2 - aq^2 = a(p + q)(p - q)$ B1
Gradient $PQ = \frac{2}{p + q}$ (c.a.o.) B1

(ii) As the point Q approaches P, PQ becomes a tangent
$\lim_{q \to p}$ (gradient PQ) $= \frac{2}{2p} = \frac{1}{p}$. E1

2016

5. (*a*) candidate's x-derivative = $-3t^{-2}$ (o.e.)
candidate's y-derivative = 4 (at least one term correct)
and use of
$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1
$\frac{dy}{dx} = \frac{4}{-3t^{-2}}$ or $-\frac{4t^2}{3}$ (c.a.o.) A1
Equation of tangent at P: $y - 4p = -\frac{4p^2}{3}\left(x - \frac{3}{p}\right)$
(f.t. candidate's expression for $\frac{dy}{dx}$) m1
Equation of tangent at P: $3y = -4p^2x + 24p$
(intermediary line required in order to be convincing) A1

(*b*) Substituting $x = 1$, $y = 9$ in equation of tangent M1
$4p^2 - 24p + 27 = 0$ A1
$p = \frac{9}{2}, \frac{3}{2}$ (both values, c.a.o.) A1
Points are ($^2/_3$, 18), (2, 6) (f.t. candidate's values for p) A1

2017

6. (*a*) candidate's x-derivative $= 2at$

candidate's y-derivative $= 3bt^2$ (at least one term correct) B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{3bt}{2a}$ (o.e.) (c.a.o.) A1

Equation of tangent at P: $y - bp^3 = \frac{3bp}{2a}(x - ap^2)$

(f.t. candidate's expression for $\frac{dy}{dx}$) m1

$2ay = 3bpx - abp^3$ (convincing) A1

(*b*) Substituting $4a$ for x and $8b$ for y in equation of tangent M1

$16ab = 12abp - abp^3 \Rightarrow p^3 - 12p + 16 = 0$ (convincing) A1

$(p - 2)(p^2 + 2p - 8) = 0$ M1

$(p - 2)(p - 2)(p + 4) = 0$ A1

$p = 2$ corresponds to $(4a, 8b) \Rightarrow p = -4$ (c.a.o.) A1