C4 Parametric Differentiation Questions

Specimen

5. A curve C has parametric equations $x = at^2$, $y = 2at$. Show that the equation of the normal to C at the point P, whose parameter is p, is

$$px + y - 2ap - ap^3 = 0.$$

The normal to C at P meets the x-axis at Q. The perpendicular from P to the x-axis meets the x-axis at R. Find the length of QR. [8]

2005 Summer

6. The parametric equations of the curve C are

$$x = 2t + 1, \quad y = t^2 + 3.$$

(a) Show that the tangent to C at the point P with parameter p has equation

$$px - y = p^2 + p - 3.$$ [4]

(b) The tangent to C at the point P passes through the point $(2, -3)$. Given that the point P is in the second quadrant, find the equation of the tangent. [4]

2006 Summer

6. The parametric equations of the curve C are

$$x = \frac{1}{t}, \quad y = t^2.$$

(a) Show that the tangent to C at the point P with parameter p has equation

$$y + 2p^3x - 3p^2 = 0.$$ [4]

(b) The tangent to C at the point P intersects the x-axis at A and the y-axis at B. Show that $PB = 2PA$. [5]

2007 Summer

6. The parametric equations of the curve C are $x = 2t$, $y = t^2$.

(a) Show that the normal to C at the point P with parameter p has equation

$$x + py = p^3 + 2p. \qquad [4]$$

(b) The normal to C at the point P intersects the x-axis at A and the y-axis at B. Given that O is the origin and $OA = 2OB$, find the value of p. [4]

2008 Summer

5. The parametric equations of the curve C are $x = 4\sin t$, $y = \cos 2t$.

(a) Find $\frac{dy}{dx}$, simplifying your answer as much as possible. [6]

(b) Show that the equation of the tangent to C at the point P with parameter p is

$$x\sin p + y = 1 + 2\sin^2 p. \qquad [3]$$

2009 Summer

5. The parametric equations of the curve C are $x = t^2$, $y = t^3$. The point P has parameter p.

(a) Show that the equation of the tangent to C at the point P is $3px - 2y = p^3$. [4]

(b) The tangent to C at the point P intersects C again at the point $Q(q^2, q^3)$. Given that $p = 2$, show that q satisfies the equation $q^3 - 3q^2 + 4 = 0$ and determine the value of q. [5]

2010 Summer

6. The parametric equations of the curve C are

$$x = \frac{2}{t}, \; y = 4t.$$

(a) Show that the tangent to C at the point P with parameter p has equation

$$y = -2p^2x + 8p. \qquad [4]$$

(b) The tangent to C at the point P passes through the point $(2, 3)$. Show that P can be one of two points. Find the coordinates of each of these two points. [4]

2011 Summer

4. The curve C has the parametric equations

$$x = 3\cos t,\ y = 4\sin t.$$

The point P lies on C and has parameter p.

(a) Show that the equation of the tangent to C at the point P is

$$(3\sin p)y + (4\cos p)x - 12 = 0.$$ [5]

(b) The tangent to C at the point P meets the x-axis at the point A and the y-axis at the point B. Given that $p = \frac{\pi}{6}$,

(i) find the coordinates of A and B,

(ii) show that the exact length of AB is $2\sqrt{19}$. [4]

2012 Summer

6. The parametric equations of the curve C are $x = t^2$, $y = 2t$.

(a) Show that the normal to C at the point P with parameter p has equation

$$y + px = p^3 + 2p.$$ [5]

(b) The normal to C at the point P intersects C again at the point with parameter 3.

(i) Show that $p^3 - 7p - 6 = 0$.

(ii) Hence show that P can be one of two points. Find the coordinates of each of these two points. [6]

2013 Summer

6. The curve C has the parametric equations

$$x = at,\ y = \frac{b}{t},$$

where a, b are positive constants.
The point P lies on C and has parameter p.

(a) Show that the equation of the tangent to C at the point P is

$$ap^2y + bx - 2abp = 0.$$ [5]

(b) The tangent to C at the point P meets the x-axis at the point A and the y-axis at the point B. Find the area of triangle AOB, where O denotes the origin. Give your answer in its simplest form. [3]

(c) The point D has coordinates $(2a, b)$. Show that there is no point P on C such that the tangent to C at the point P passes through D. [3]

2014 Summer

6. The curve C has the parametric equations $x = 2t$, $y = 5t^3$. The point P lies on C and has parameter p.

(a) Show that the equation of the tangent to C at the point P is

$$2y = 15p^2x - 20p^3.$$

[4]

(b) The tangent to C at the point P intersects C again at the point $Q(2q, 5q^3)$. Given that $p = 1$, show that q satisfies the equation

$$q^3 - 3q + 2 = 0.$$

Hence find the value of q. [5]

2015

6. The parametric equations of the curve C are $x = at^2$, $y = 2at$, where a is a positive constant. The points P and Q lie on C and have parameters p and q respectively.

(a) Simplifying your answer in each case, find

(i) the gradient of the tangent to C at the point P,

(ii) the equation of the tangent to C at the point P. [4]

(b) (i) Find an expression, in its simplest form, for the gradient of the line PQ.

(ii) Explain how you could use the answer of *(b)*(i) to derive the gradient of the tangent to C at the point P. [4]

2016

5. The parametric equations of the curve C are

$$x = \frac{3}{t}, \quad y = 4t.$$

(a) Show that the tangent to C at the point P with parameter p has equation

$$3y = -4p^2x + 24p.$$

[4]

(b) The tangent to C at the point P passes through the point (1, 9). Show that P can be one of two points. Find the coordinates of each of these two points. [4]

2017

6. The curve C has the parametric equations $x = at^2$, $y = bt^3$, where a, b are positive constants. The point P lies on C and has parameter p.

(a) Show that the equation of the tangent to C at the point P is

$$2ay = 3bpx - abp^3.$$ [5]

(b) The tangent to C at the point P intersects C again at the point with coordinates $(4a, 8b)$. Show that p satisfies the equation

$$p^3 - 12p + 16 = 0.$$

Hence find the value of p. [5]