C4 Partial Fractions Answers

Specimen

4. *(a)* Let $\frac{3x^2+2x+1}{x^2(x-1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$ — M1 (correct form)

$\therefore 3x^2 + 2x + 1 \equiv Ax(x-1) + B(x-1) + Cx^2$ — m1 (clearing fraction and attempt to find constants)

so that $A = -3$, $B = -1$, $C = 6$ — A2 (3 constants)
A1 (2 constants)

(b) $\int \frac{3x^2+2x+1}{x^2(x-1)}\,dx = -3\ln|x| + \frac{1}{x}$ — B1, B1

$+ 6\ln|x-1|$ — B1

($| \quad |$ may be omitted)

2005 Summer

1. (a) Let $\frac{8x^2+x-5}{(2x-1)^2(x+2)} \equiv \frac{A}{2x-1} + \frac{B}{(2x-1)^2} + \frac{C}{(x+2)}$ — M1

$\therefore\ 8x^2 + x - 5 \equiv A(2x-1)(x+2) + B(x+2) + C(2x-1)^2$ — M1

$x = \frac{1}{2}$ B = –1, $x = -2$, C = 1 — A1

Equate coefficients of x^2: 2A + 4C = 8
A = 2 — A1

(b) $\int \frac{2}{2x-1}\,dx - \int \frac{1}{(2x-1)^2}\,dx + \int \frac{1}{x+2}\,dx$

$= \ln|2x-1| + \frac{1}{2(2x-1)} + \ln|x+2|$ (+ C) — B1, B1, B1

(no need for modulus)

[7]

2006 Summer

1. *(a)* Let $\frac{2x^2+4}{(x-2)^2(x+4)} \equiv \frac{A}{x+4} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$ — M1 (Correct form)

$2x^2 + 4 \equiv A(x-2)^2 + B(x-2) + C(x+4)$ — M1 (correct clearing of fractions and attempt to substitute)

$\underline{x = 2}$ $\quad 12 = C(6), \quad C = 2$

$\underline{x = -4}$ $\quad 36 = A(36), \quad A = 1$ — A1 (two constants, C.A.O.)

$\underline{\text{Coefft of } x^2}$ $\quad 2 = A + B, \quad B = 1$ — A1 (third constant, F.T. one slip)

(b) $f'(x) = \frac{-1}{(x+4)^2} - \frac{1}{(x-2)^2}$ — B1 (first two terms)

$- \frac{4}{(x-2)^3}$ — B1 (third term)

$f'(0) = -\frac{1}{16} - \frac{1}{4} + \frac{4}{8} = \frac{3}{16}$ (o.e.) — B1 (C.A.O.)

7

2007 Summer

1. *(a)* Let $\frac{x+3}{x^2(x-1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$ — M1 (correct form)

$\therefore x + 3 \equiv Ax(x-1) + B(x-1) + Cx^2$ — M1 (clearing fractions and attempt to solve)

$\underline{x = 1}$ $\quad 4 = C, \quad C = 4$ — A1 (2 constants)

$\underline{x = 0}$ $\quad 3 = -B, \quad B = -3$ — A1 (other constant

Equate coefficients of x^2 $\quad 0 = A + C, \quad A = -4$ — (F.T. if 2 Ms scored)

No need for display

(b) $\int -\frac{4}{x}dx - \int \frac{3}{x^2}dx + \int \frac{4}{x-1}dx$

$= -4\,ln(x) + \frac{3}{x} + 4\,ln\,|x-1|$ (o.e.) — B1, B1 (two logs)

$(+\,C)$

6

2008 Summer

1. *(a)* Let $\dfrac{1}{x^2(2x-1)} \equiv \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{2x-1}$ — M1 (Correct form)

$1 \equiv Ax(2x-1) + B(2x-1) + Cx^2$ — M1 (correct clearing and attempt to substitute)

$\underline{x=0}$ $\quad 1 = B(-1) \quad \therefore B = -1$

$\underline{x=\tfrac{1}{2}}$ $\quad 1 = C\tfrac{1}{4} \quad \therefore C = 4$ — A1 (2 constants C.A.O.)

$\underline{x^2}$ $\quad 0 = 2A + C \quad \therefore A = -2$ — A1 (third constant, F.T. one slip)

(no need for display)

(b) $\displaystyle\int \left(\frac{-2}{x} - \frac{1}{x^2} + \frac{4}{2x-1} \right) dx$

$= -2\ln|x| + \dfrac{1}{x} + 2\ln|2x-1|$ — B1,B1.B1

$(+C)$

7

2009 Summer

1. *(a)* $\dfrac{3x}{(1+x)^2(2+x)} \equiv \dfrac{A}{1+x} + \dfrac{B}{(1+x)^2} + \dfrac{C}{2+x}$ — (correct form) M1

$3x \equiv A(1+x)(2+x) + B(2+x) + C(1+x)^2$ — (correct attempt to clear fractions and substitute for x) M1

$x = -1$ $\quad -3 = B(1)$

$B = -3$

$x = -2$ $\quad -6 = C(-1)^2$ — (2 constants) A1

$C = -6$

x^2 $\quad 0 = A + C$

$A = 6$ — (3^{rd} constant) A1 (F.T. one slip)

(b) $\displaystyle\int_0^1 \left(\frac{6}{1+x} - \frac{3}{(1+x)^2} - \frac{6}{2+x} \right) dx$

$= \left[6\ln(1+x) + \dfrac{3}{1+x} - 6\ln(2+x) \right]_0^1$ — $\left(\dfrac{3}{1+x}\right)$ B1

(F.T. candidate's equivalent work) — (logs) B1, B1

$= 6\ln 2 + \dfrac{3}{2} - 6\ln 3 - 6\ln 1 - 3 + 6\ln 2$

$\approx 0{\cdot}226$ — (must be at least 3 decimal places) C.A.O. B1

2010 Summer

1. (*a*) $f(x) \equiv \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$ (correct form) M1

$8 - x - x^2 \equiv A(x-2)^2 + Bx(x-2) + Cx$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 2, C = 1, B = -3$ (2 coefficients, c.a.o.) A1

(third coefficient, f.t. one slip in enumeration of other 2 coefficients) A1

(*b*) $f'(x) = \frac{-2}{x^2} + \frac{3}{(x-2)^2} - \frac{2}{(x-2)^3}$ (at least one of first two terms) B1

(third term) B1

(f.t. candidates values for A, B, C)

$f'(1) = 3$ (c.a.o.) B1

2011 Summer

1. (*a*) $f(x) \equiv \frac{A}{(x+2)^2} + \frac{B}{(x+2)} + \frac{C}{(x-3)}$ (correct form) M1

$x^2 + x + 13 \equiv A(x-3) + B(x+2)(x-3) + C(x+2)^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -3, C = 1, B = 0$ (all three coefficients correct) A2

(at least one coefficient correct) A1

(*b*) $\int f(x)\,dx = \frac{3}{(x+2)} + \ln(x-3)$ B1 B1

(f.t. candidates values for A, B, C)

$\int_6^7 f(x)\,dx = \left[\frac{3}{9} - \frac{3}{8}\right] - [\ln 4 - \ln 3] = 0{\cdot}246(015405)$ (c.a.o.) B1

Note: Answer only with no working earns 0 marks

2012 Summer

1. (*a*) $f(x) \equiv \dfrac{A}{(x+1)} + \dfrac{B}{(x-2)} + \dfrac{C}{(x-2)^2}$ (correct form) M1

$11 + x - x^2 \equiv A(x-2)^2 + B(x+1)(x-2) + C(x+1)$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 1, C = 3, B = -2$ (2 correct coefficients) A1

(third coefficient, f.t. one slip in enumeration of other 2 coefficients) A1

(*b*) $f'(x) = -\dfrac{1}{(x+1)^2} + \dfrac{2}{(x-2)^2} - \dfrac{6}{(x-2)^3}$ (o.e.)

(f.t. candidate's values for A, B, C)

(at least one of the first two terms) B1

(third term) B1

$f'(0) = {}^1/_4$ (c.a.o.) B1

2013 Summer

1. (*a*) $f(x) \equiv \dfrac{A}{x^2} + \dfrac{B}{x} + \dfrac{C}{(x+2)}$ (correct form) M1

$6 + x - 9x^2 \equiv A(x+2) + Bx(x+2) + Cx^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 3, C = -8, B = -1$ (all three coefficients correct) A2

If A2 not awarded, award A1 for at least one correct coefficient

(*b*) (i) $f'(x) = \dfrac{-6}{x^3} + \dfrac{1}{x^2} + \dfrac{8}{(x+2)^2}$ (o.e.)

(f.t. candidate's values for A, B, C)

(first term) B1

(at least one of last two terms) B1

(ii) $f'(2) = 0 \Rightarrow$ stationary value when $x = 2$ (c.a.o.) B1

2014 Summer

2. (*a*) $f(x) \equiv \dfrac{A}{(x+1)^2} + \dfrac{B}{(x+1)} + \dfrac{C}{(x-4)}$ (correct form) M1

$5x^2 + 7x + 17 \equiv A(x-4) + B(x+1)(x-4) + C(x+1)^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -3,\ C = 5,\ B = 0$ (all three coefficients correct) A2

(**If A2 not awarded**, award A1 for either 1 or 2 correct coefficients)

(*b*) $\dfrac{5x^2 + 9x + 9}{(x+1)^2(x-4)} = \dfrac{5x^2 + 7x + 17}{(x+1)^2(x-4)} + \dfrac{2}{(x+1)^2}$ M1

$\dfrac{5x^2 + 9x + 9}{(x+1)^2(x-4)} = \dfrac{-1}{(x+1)^2} + \dfrac{5}{(x-4)}$

(f.t. candidates values for *A*, *B*, *C*) A1

2015

1. (*a*) $f(x) \equiv \dfrac{A}{(x+3)^2} + \dfrac{B}{(x+3)} + \dfrac{C}{(x-1)}$ (correct form) M1

$2x^2 + 5x + 25 \equiv A(x-1) + B(x+3)(x-1) + C(x+3)^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -7,\ C = 2,\ B = 0$ (all three coefficients correct) A2

If A2 not awarded, award A1 for at least one correct coefficient

(*b*) $\displaystyle\int f(x)\,dx = \frac{7}{(x+3)} + 2\ln(x-1)$ B1 B1

(f.t. candidate's values for *A*, *B*, *C*)

$\displaystyle\int_3^{10} f(x)\,dx = \left[\frac{7}{13} + 2\ln 9\right] - \left[\frac{7}{6} + 2\ln 2\right] = 2{\cdot}38$ (c.a.o.) B1

Note: Answer only with no working earns 0 marks

2016

1. (*a*) $f(x) \equiv \frac{A}{(2x-1)} + \frac{B}{(x-3)^2} + \frac{C}{(x-3)}$ (correct form) M1

$17 + 4x - x^2 \equiv A(x-3)^2 + B(2x-1) + C(x-3)(2x-1)$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 3, B = 4, C = -2$ (all three coefficients correct) A2

If A2 not awarded, award A1 for at least one correct coefficient

(*b*) $f'(x) = -\frac{6}{(2x-1)^2} - \frac{8}{(x-3)^3} + \frac{2}{(x-3)^2}$ (o.e.)

(f.t. candidate's derived values for A, B, C)

(second term) B1

(**both** the first and third terms) B1

2017

1. (*a*) $f(x) \equiv \frac{A}{(x-1)^2} + \frac{B}{(x-1)} + \frac{C}{(x+4)}$ (correct form) M1

$8x^2 + 7x - 25 \equiv A(x+4) + B(x-1)(x+4) + C(x-1)^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -2, C = 3, B = 5$ (all three coefficients correct) A2

(**If A2 not awarded**, award A1 for either 1 or 2 correct coefficients)

(*b*) $\frac{9x^2 + 5x - 24}{(x-1)^2(x+4)} = \frac{8x^2 + 7x - 25}{(x-1)^2(x+4)} + \frac{x^2 - 2x + 1}{(x-1)^2(x+4)}$ M1

$\frac{x^2 - 2x + 1}{(x-1)^2(x+4)} = \frac{1}{x+4}$ A1

$\frac{9x^2 + 5x - 24}{(x-1)^2(x+4)} = \frac{-2}{(x-1)^2} + \frac{5}{(x-1)} + \frac{4}{(x+4)}$

(f.t. candidate's values for A, B, C) A1