C4 Proof by Contradiction Answers

2005 Summer

10. $x^2 - 10x + 25 < 0$ B1

$(x - 5)^2 < 0$ B1

$x - 5$ not real B1

(x not real)

Contradiction B1

so $x + \frac{25}{x} \geq 10$

[4]

2006 Summer

11. $4k^2 = 2b^2$ B1

$b^2 = 2k^2$
b^2 has a factor 2 } one or other of these statements B1

$\therefore b$ has a factor 2 B1 (if and only if previous B gained)

a and b have a common factor
Contradiction B1 (depends upon previous B being gained)

($\therefore \sqrt{2}$ is irrational)

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2007 Summer

10. $3n + 2n^3 = 3(2k) + 2(2k)^3$

$= 6k + 16k^3$

$= 2(3k + 8k^3)$ B1 (either $2x$ ( ) or even + even = even)

which is even
Contradiction B1
(Thus n is odd)

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2008 Summer

10.	$(x^2+49<14x)$	
	$x^2-14x+49<0$	B1
	$(x-7)^2<0$	B1
	$x-7$ is not real	B1
	contradiction	B1 (accept impossibility)
	$\left(\therefore x+\dfrac{49}{x}\geq 14 \quad \textit{for all real and positive } x\right)$	
		4

2009 Summer

10.	$9k^2=3b^2$	B1
	$b^2=3k^2$	B1
	(b^2 has a factor 3)	
	b has a factor 3	B1
	a and b have a common factor – contradiction (must mention contradiction)	B1
	($\sqrt{3}$ is irrational)	

2010 Summer

10.	Assume that positive real numbers a, b exist such that $a+b<2\sqrt{ab}$.	
	Squaring both sides we have: $(a+b)^2<4ab \Rightarrow a^2+b^2+2ab<4ab$	B1
	$a^2+b^2-2ab<0 \Rightarrow (a-b)^2<0$	B1
	This contradicts the fact that a, b are real and thus $a+b\geq 2\sqrt{ab}$	B1

2011 Summer

10.	Assume that there is a real and positive value of x such that $4x+\dfrac{9}{x}<12$	
	$4x^2-12x+9<0$	B1
	$(2x-3)^2<0$	B1
	This contradicts the fact that x is real and thus $4x+\dfrac{9}{x}\geq 12$	B1

2012 Summer

10. $a^2 = 5b^2 \Rightarrow (5k)^2 = 5b^2 \Rightarrow b^2 = 5k^2$ B1

$\therefore$ 5 is a factor of b^2 and hence 5 is a factor of b B1

$\therefore$ a and b have a common factor, which is a contradiction to the original assumption B1

2013 Summer

10. Assume that there is a real value of x such that

$$(5x-3)^2 + 1 < (3x-1)^2.$$

$25x^2 - 30x + 9 + 1 < 9x^2 - 6x + 1 \Rightarrow 16x^2 - 24x + 9 < 0$ B1

$(4x-3)^2 < 0$ B1

This contradicts the fact that x is real and thus $(5x-3)^2 + 1 \geq (3x-1)^2$. B1

2014 Summer

10. Squaring both sides we have

$1 + 2\sin\theta\cos\theta > 2$ B1

$\sin 2\theta > 1$ B1

Contradiction, since the sine of any angle ≤ 1 B1

2015

10. Assume that 4 is a factor of $a + b$.

Then there exists an integer c such that $a + b = 4c$.

Similarly, there exists an integer d such that $a - b = 4d$. B1

Adding, we have $2a = 4c + 4d$. B1

Therefore $a = 2c + 2d$, an even number, which contradicts the fact that a is odd. B1

2016

10. Assume that there is a real value of x such that

$$\left|x+\frac{1}{x}\right|<2$$

Then squaring both sides, we have:

$x^2+\frac{1}{x^2}+2<4$ B1

$x^2+\frac{1}{x^2}-2<0$ B1

$\left(x-\frac{1}{x}\right)^2<0$, which is impossible since the square of a real number cannot be negative B1

Alternative Mark Scheme

Assume that there is a real value of x such that

$$\left|x+\frac{1}{x}\right|<2$$

Then squaring both sides, we have:

$x^2+\frac{1}{x^2}+2<4$ B1

$x^4-2x^2+1<0$ B1

$(x^2-1)^2<0$, which is impossible since the square of a real number cannot be negative B1

2017

10. $a^2=7b^2 \Rightarrow (7k)^2=7b^2 \Rightarrow b^2=7k^2$ B1

$\therefore$ 7 is a factor of b^2 and hence 7 is a factor of b B1

$\therefore$ a and b have a common factor, which is a contradiction to the original assumption B1