C4 Proof by Contradiction Questions

2005 Summer

10. Complete the following proof by contradiction to show that $x + \frac{25}{x} \geqslant 10$ when x is real and positive.

Assume that $x + \frac{25}{x} < 10$ *, when x is real and positive.*

Since x is positive, multiplication of both sides of the inequality by x gives $x^2 + 25 < 10x$. [4]

2006 Summer

11. Complete the following proof by contradiction to show that $\sqrt{2}$ is irrational.

Assume that $\sqrt{2}$ *is rational. Then* $\sqrt{2}$ *may be written in the form* $\frac{a}{b}$ *, where a and b are positive integers having no common factor.*

$\therefore$ $a^2 = 2b^2$.

$\therefore$ a^2 *has a factor 2.*

$\therefore$ *a has a factor 2 so that* $a = 2k$,

where k is an integer. [4]

2007 Summer

10. Complete the following proof by contradiction to show that, if n is a positive integer and $3n + 2n^3$ is odd, then n is odd. [2]

It is given that $3n + 2n^3$ *is odd.*
Assume that n is even so that $n = 2k$.

2008 Summer

10. Prove by contradiction the following proposition.

When x is real and positive,

$$x + \frac{49}{x} \geqslant 14 \ .$$

The first line of the proof is given below.

Assume that there is a positive and real value of x such that

$$x + \frac{49}{x} < 14 \ . \qquad [4]$$

2009 Summer

10. Complete the following proof by contradiction to show that $\sqrt{3}$ is irrational.

Assume that $\sqrt{3}$ *is rational. Then* $\sqrt{3}$ *may be written in the form* $\frac{a}{b}$ *where a and b are integers having no common factors.*

$\therefore\ a^2 = 3b^2$.
$\therefore\ a^2$ has a factor 3.
$\therefore\ a$ has a factor 3 so that $a = 3k$, where k is an integer. [4]

2010 Summer

10. Prove by contradiction the following proposition.

If a, b are positive real numbers, then $a + b \geqslant 2\sqrt{ab}$.

The first line of the proof is given below.

Assume that positive real numbers a, b exist such that $a + b < 2\sqrt{ab}$. [3]

2011 Summer

10. Prove by contradiction the following proposition.

When x is real and positive,

$$4x + \frac{9}{x} \geqslant 12.$$

The first line of the proof is given below.

Assume that there is a positive and real value of x such that

$$4x + \frac{9}{x} < 12. \qquad [3]$$

2012 Summer

10. Complete the following proof by contradiction to show that $\sqrt{5}$ is irrational.

Assume that $\sqrt{5}$ is rational. Then $\sqrt{5}$ may be written in the form $\frac{a}{b}$, where a, b are integers having no common factors.

$\therefore$ *$a^2 = 5b^2$.*
$\therefore$ *a^2 has a factor 5.*
$\therefore$ *a has a factor 5 so that $a = 5k$, where k is an integer.* [3]

2013 Summer

10. Prove by contradiction the following proposition.

When x is real,

$$(5x - 3)^2 + 1 \geqslant (3x - 1)^2.$$

The first line of the proof is given below.

Assume that there is a real value of x such that

$$(5x - 3)^2 + 1 < (3x - 1)^2.$$ [3]

2014 Summer

10. Complete the following proof by contradiction to show that

$$\sin\theta + \cos\theta \leqslant \sqrt{2}$$

for all values of θ.

Assume that there is a value of θ for which $\sin\theta + \cos\theta > \sqrt{2}$.
Then squaring both sides, we have:

[3]

END OF PAPER

2015

10. Prove by contradiction the following proposition.

> If a and b are odd integers such that 4 is a factor of $a - b$,
> then 4 is **not** a factor of $a + b$.

The first lines of the proof are given below.

> *Assume that 4 is a factor of $a + b$.*
> *Then there exists an integer c such that $a + b = 4c$.* [3]

2016

10. Prove by contradiction the following proposition.

> When x is real and $x \neq 0$,
> $$\left|x + \frac{1}{x}\right| \geqslant 2.$$

The first two lines of the proof are given below.

> *Assume that there is a real value of x such that*
> $$\left|x + \frac{1}{x}\right| < 2.$$
> *Then squaring both sides, we have:* [3]

2017

10. Complete the following proof by contradiction to show that $\sqrt{7}$ is irrational.

> *Assume that $\sqrt{7}$ is rational. Then $\sqrt{7}$ may be written in the form $\frac{a}{b}$,*
> *where a, b are integers having no factors in common.*
> $\therefore a^2 = 7b^2$.
> $\therefore a^2$ *has a factor 7.*
> $\therefore a$ *has a factor 7 so that $a = 7k$, where k is an integer.* [3]