Trig Topic Paper Answers

Specimen

2.	*(a)*	Substitution of appropriate value of θ Convincing conclusion	B1 A1
	(b)	$3(1 - 2\sin^2\theta) = 1 - \sin\theta$	M1 (correct elimination of $\cos^2\theta$)
		$\therefore 6\sin^2\theta - \sin\theta - 2 = 0$	M1 (Standard form of quadratic and attempt to solve)
		$\therefore (3\sin\theta - 2)(2\sin\theta + 1) = 0$	
		so that $\sin\theta = \frac{2}{3}, -\frac{1}{2}$	A1
		$\therefore \theta = 41{\cdot}8^\circ, 138{\cdot}2^\circ, 210^\circ, 330^\circ$	B1 ($41{\cdot}8^\circ$, $138{\cdot}2^\circ$) B1 (210°) B1 (330°)
3.		$R\cos\alpha = 5,\ R\sin\alpha = 4$	B1 (both)
		$R = \sqrt{41},\ \alpha = 38{\cdot}66^\circ$	M1 (any method of funding R or α) A1 (R) A1 ($38{\cdot}7^\circ$)
		$\therefore \sin(\theta + 38{\cdot}7^\circ) = \frac{3}{\sqrt{41}}$	M1
		$\therefore \theta + 38{\cdot}66^\circ = 152{\cdot}06^\circ, 387{\cdot}94^\circ$	A1 (any value)
		$\therefore \theta = 113{\cdot}4^\circ, 349{\cdot}3^\circ$	A1 (both)

2005

4. (a) $2\sin\theta\cos\theta = \cos\theta$ — M1

$\cos\theta = 0,\ \sin\theta = \frac{1}{2}$

$\theta = 90°, 270°, 30°, 150°$ — A3 (–1 for each omission) (–1 for each additional value)

(b) Using $R\sin(\theta + \alpha)$ with $R = \sqrt{17}$, $\alpha = 14.04°$ — M1 A1 A1

$\sin(\theta + 14.04°) = \frac{2}{\sqrt{17}}$

$\theta + 14.04° = 29.02°, 150.98°$ — B1 (1 value)

$\theta = 14.98°, 136.94°$ — B1, B1

[10]

2006

3. $2 + 3(2\cos^2\theta - 1) = \cos\theta$ — M1 (correct substitution for $\cos 2\theta$)

$6\cos^2\theta - \cos\theta - 1 = 0$ — M1 (correct method of solution, $(a\cos\theta + b)(\cos\theta + d)$ with ac = coefft of $\cos^2\theta$, bd = constant term, or correct formula)

$(3\cos\theta + 1)(2\cos\theta - 1) = 0 \quad \cos\theta = -\frac{1}{3}, \frac{1}{2}$ — A1

$\theta = 109{\cdot}5°, 250{\cdot}5°, 60°, 300°$ — B1 (109·5°) B1 (250·5°) B1 (60°, 300°)

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2007

3. $4\cos x + 2\sin x = R\cos(x - \alpha)$ — M1 (for $R\cos(x \pm \alpha)$)

$R\cos\alpha = 4,\ R\sin\alpha = 2$ — B1 ($\sqrt{20}$)

$R = \sqrt{20},\ \alpha = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$ — A1 (correct α for given presentation)

$\cos(x - 26.6°) = \frac{3}{\sqrt{20}}$ — M1 ($\cos(x \pm 26.6°) = \frac{3}{\sqrt{20}}$)

$x - 26.6° = 47.9°, 312.1°$ — A1 (for one value) (C.A.O.)

$x = 74.4°, 338.7°$ (C.A.O.) — (accept 74, 75, 338°, 339°) A1, A1

2008

3. *(a)* $R\sin\alpha = 2$, $R\cos\alpha = 3$ — B1 ($R = \sqrt{13}$)

$R = \sqrt{13}$, $\alpha = \tan^{-1}\left(\frac{2}{3}\right) = 33.7° or 34°$ — M1 (correct method for α)

A1 ($\alpha = 34°$)

(b) $\cos(x - 33.7°) = \frac{1}{\sqrt{13}}$

$x - 33.7° = 73.9°, 286.1°$ — B1 (one value)

$x = 107.6°, 319.8°$ — B1, B1

2009

3. *(a)* $R = 2$ — B1

$\tan\alpha = \sqrt{3}$, $\alpha = 60°$ — (any method) M1

A1

(b) $2\cos(\theta - 60°) = 1$

$\cos(\theta - 60°) = \frac{1}{2}$ — (F.T. R and α) M1

$\theta - 60° = -60°, 60°, 300°$ — (one value) A1

$\theta = 0°, 120°, 360°$ — (A2 for 3 answers, A1 for 2 answers A0 for 1 answer, lose 1 for more than 3 answers) A2

2010

3. (*a*) $2(2\cos^2\theta - 1) = 9\cos\theta + 7$

(correct use of $\cos 2\theta = 2\cos^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = coefficient of $\cos^2\theta$ and $b \times d$ = constant — m1

$4\cos^2\theta - 9\cos\theta - 9 = 0 \Rightarrow (4\cos\theta + 3)(\cos\theta - 3) = 0$

$\Rightarrow \cos\theta = -\frac{3}{4}$, $(\cos\theta = 3)$ (c.a.o.) — A1

$\theta = 138{\cdot}59°, 221{\cdot}41°$ — B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range from $4\cos\theta + 3 = 0$, ignore roots outside range.
$\cos\theta = -$, f.t. for 2 marks, $\cos\theta = +$, f.t. for 1 mark

(*b*) (i) $R = 13$ B1

Correctly expanding $\sin(x - \alpha)$ and using either $13\cos\alpha = 5$ **or** $13\sin\alpha = 12$ **or** $\tan\alpha = \frac{12}{5}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 67{\cdot}38°$ (c.a.o) A1

(ii) Least value of $\dfrac{1}{5\sin x - 12\cos x + 20} = \dfrac{1}{13 \times (\pm 1) + 20}$.

(f.t. candidate's value for R) M1

Least value $= \dfrac{1}{33}$ (f.t. candidate's value for R) A1

Corresponding value for $x = 157{\cdot}38°$ (o.e.)

(f.t. candidate's value for α) A1

2011

3. (*a*) $\dfrac{2\tan x}{1 - \tan^2 x} = 4\tan x$ (correct use of formula for $\tan 2x$) M1

$\tan x = 0$ A1

$2\tan^2 x - 1 = 0$ A1

$x = 0°, 180°$ (both values) A1

$x = 35{\cdot}26°, 144{\cdot}74°$ (both values) A1

(*b*) $R = 25$ B1

Expanding $\cos(\theta - \alpha)$ and using either $25\cos\alpha = 7$ **or** $25\sin\alpha = 24$ **or** $\tan\alpha = \frac{24}{7}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 73{\cdot}74°$ (c.a.o.) A1

$\cos(\theta - \alpha) = \dfrac{16}{25} = 0{\cdot}64$ (f.t. candidate's value for R) B1

$\theta - \alpha = 50{\cdot}21°, -50{\cdot}21°$

(at least one value, f.t. candidate's value for R) B1

$\theta = 23{\cdot}53°, 123{\cdot}95°$ (c.a.o.) B1

2012

3. (*a*) $4(1 - 2\sin^2\theta) = 1 - 2\sin\theta$. (correct use of $\cos 2\theta = 1 - 2\sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant m1

$8\sin^2\theta - 2\sin\theta - 3 = 0 \Rightarrow (4\sin\theta - 3)(2\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = \frac{3}{4}$, $\sin\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 48{\cdot}59°, 131{\cdot}41°$ B1

$\theta = 210°, 330°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks
$\sin\theta = +, +$, f.t. for 1 mark

(*b*) (i) $R = 17$ B1

Correctly expanding $\sin(x + \alpha)$ and using either $17\cos\alpha = 8$ or $17\sin\alpha = 15$ or $\tan\alpha = \frac{15}{8}$ to find α
(f.t. candidate's value for R) M1

$\alpha = 61{\cdot}93°$ (c.a.o) A1

(ii) $\sin(x + \alpha) = \frac{11}{17}$ (f.t. candidate's value for R) B1

$x + 61{\cdot}93° = 40{\cdot}32°, 139{\cdot}68°, 400{\cdot}32°$,
(at least one value on R.H.S., f.t. candidate's values for α and R) B1

$x = 77{\cdot}75°, 338{\cdot}39°$ (c.a.o.) B1

(iii) Greatest possible value for k is 17 since greatest possible value for sin is 1 (f.t. candidate's value for R) E1

2013

3. (*a*) $8(2\cos^2\theta - 1) + 6 = \cos^2\theta + \cos\theta$

(correct use of $\cos 2\theta = 2\cos^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$15\cos^2\theta - \cos\theta - 2 = 0 \Rightarrow (5\cos\theta - 2)(3\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \frac{2}{5}, \quad \cos\theta = -\frac{1}{3}$, (c.a.o.) A1

$\theta = 66{\cdot}42°, 293{\cdot}58°$ B1

$\theta = 109{\cdot}47°, 250{\cdot}53°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\cos\theta = +, -$, f.t. for 3 marks, $\cos\theta = -, -$, f.t. for 2 marks

$\cos\theta = +, +$, f.t. for 1 mark

(*b*) $R = 4$ B1

Correctly expanding $\cos(\theta + \alpha)$, correctly comparing coefficients and using either $4\cos\alpha = \sqrt{15}$ **or** $4\sin\alpha = 1$ **or** $\tan\alpha = \frac{1}{\sqrt{15}}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 14{\cdot}48°$ (c.a.o.) A1

$\cos(\theta + 14{\cdot}48°) = \frac{3}{4} = 0{\cdot}75$

(f.t. candidate's values for R, α, $0° < \alpha < 90°$) B1

$\theta + 14{\cdot}48° = 41{\cdot}41°, 318{\cdot}59°$

(at least one value, f.t. candidate's values for R, α, $0° < \alpha < 90°$) B1

$\theta = 26{\cdot}93°, 304{\cdot}11°$ (c.a.o.) B1

3. (*a*)

$\frac{2\tan x}{1-\tan^2 x} = 3\cot x$ (correct use of formula for $\tan 2x$) M1

$\frac{2\tan x}{1-\tan^2 x} = \frac{3}{\tan x}$ (correct use of $\cot x = \frac{1}{\tan x}$) M1

$\tan^2 x = {}^3/_5$ (o.e.) A1

$x = 37{\cdot}76°, 142{\cdot}24°$ (both values)

(f.t. $a\tan^2 x = b$ provided both M1's are awarded) A1

(*b*) (i) $R = 29$ B1

Correctly expanding $\sin(\theta - \alpha)$ and using either $29\cos\alpha = 21$ **or** $29\sin\alpha = 20$ **or** $\tan\alpha = \frac{20}{21}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 43{\cdot}6°$ (c.a.o) A1

(ii) Greatest value of $\frac{1}{21\sin\theta - 20\cos\theta + 31} = \frac{1}{29\times(\pm1)+31}$

(f.t. candidate's value for R) M1

Greatest value $= \frac{1}{2}$ (f.t. candidate's value for R) A1

Corresponding value for $\theta = 313{\cdot}6°$ (o.e.)

(f.t. candidate's value for α) A1

2015

3. (*a*) $\dfrac{\tan x + \tan 45°}{1 - \tan x \tan 45°} = 8 \tan x$ (correct use of formula for $\tan(x + 45°)$) M1

Use of $\tan 45° = 1$ and an attempt to form a quadratic in $\tan x$ by cross multiplying and collecting terms M1

$8\tan^2 x - 7\tan x + 1 = 0$ (c.a.o.) A1

Use of a correct method to solve the candidate's derived quadratic in $\tan x$ m1

$x = 34{\cdot}8°, 10{\cdot}2°$ (both values)

(f.t. one slip in candidate's derived quadratic in $\tan x$ provided all three method marks have been awarded) A1

(*b*) (i) $R = 7$ B1

Correctly expanding $\sin(\theta - \alpha)$, correctly comparing coefficients and using either $7\cos\alpha = \sqrt{13}$ **or** $7\sin\alpha = 6$ **or** $\tan\alpha = \dfrac{6}{\sqrt{13}}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 59°$ (c.a.o) A1

(ii) $\sin(\theta - \alpha) = -\dfrac{4}{7}$

(f.t. candidate's values for R, α) B1

$\theta - 59° = -34{\cdot}85°, 214{\cdot}85°, 325{\cdot}15°,$

(at least one value, f.t. candidate's values for R, α) B1

$\theta = 24{\cdot}15°, 273{\cdot}85°$ (c.a.o.) B1

2016

4. (*a*) (i) $\dfrac{6\tan x}{1-\tan^2 x} + 16\cot^2 x = 0$ (o.e.)

(correct use of formula for tan 2*x*) M1

$\dfrac{6\tan x}{1-\tan^2 x} + \dfrac{16}{\tan^2 x} = 0$ (correct use of $\cot^2 x = \dfrac{1}{\tan^2 x}$) M1

$3\tan^3 x - 8\tan^2 x + 8 = 0$

(intermediary line required in order to be convincing) A1

(ii) $3\tan^3 x - 8\tan^2 x + 8 = (\tan x - 2)(3\tan^2 x + a\tan x + b)$

with one of *a*, *b* correct M1

$3\tan^3 x - 8\tan^2 x + 8 = (\tan x - 2)(3\tan^2 x - 2\tan x - 4)$ A1

$x = 63{\cdot}4°, 56{\cdot}9°, 139{\cdot}0°$

(rounding off errors are only penalised once) A1 A1 A1

(*b*) $R = 25$ B1

Correctly expanding $\cos(\theta + \alpha)$ and using either $25\cos\alpha = 24$ or $25\sin\alpha = 7$ **or** $\tan\alpha = \dfrac{7}{24}$ to find α

(f.t. candidate's value for *R*) M1

$\alpha = 16{\cdot}26°$ (c.a.o) A1

Use of both critical values -25 and 25

(f.t candidate's derived value for *R*) M1

$25\cos(\theta + \alpha) = k$ has no solutions if $k < -25$ or $k > 25$

(f.t candidate's derived value for *R*) A1

2017

3. (*a*) $5\cos^2\theta + 7 \times 2\sin\theta\cos\theta = 3\sin^2\theta$

(correct use of $\sin 2\theta = 2\sin\theta\cos\theta$) M1

An attempt to form a quadratic equation in $\tan\theta$ by dividing throughout by $\cos^2\theta$ and then using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ m1

$3\tan^2\theta - 14\tan\theta - 5 = 0$ (c.a.o.) A1

$\tan\theta = -\frac{1}{3}$, $\tan\theta = 5$ (c.a.o.) A1

$\theta = 161\cdot57°$ B1

$\theta = 78\cdot69°$ B1

Note: F.t. candidate's derived quadratic equation in $\tan\theta$. Do not award the corresponding B1 if the candidate gives more than one root in that particular branch. Ignore roots outside range.

(*b*) (i) $R = 4$ B1

Correctly expanding $\cos(\phi - \alpha)$ and using either $4\cos\alpha = \sqrt{5}$ or $4\sin\alpha = \sqrt{11}$ or $\tan\alpha = \frac{\sqrt{11}}{\sqrt{5}}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 56°$ (c.a.o) A1

(ii) Least value of $\frac{1}{\sqrt{5}\cos\phi + \sqrt{11}\sin\phi + 6} = \frac{1}{4 \times k + 6}$.

($k = 1$ or -1)

(f.t. candidate's value for R) M1

Least value $= \frac{1}{10}$ (f.t. candidate's value for R) A1

Corresponding value for $\phi = 56°$ (o.e.)

(f.t. candidate's value for α) A1