C4 Vector Answers

Specimen

9. *(a)*

Where the lines intersect,	
$2\mathbf{i} + \mathbf{j} + \lambda(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	M1 (attempt to equate **i, j, k** terms)
$= 2\mathbf{i} + 2\mathbf{j} + t\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	
$\therefore 2 + \lambda = 2 + \mu$, $1 + \lambda = 2 + 2\mu$, $2\lambda = t + \mu$.	A1 (correct)
Then $\lambda = \mu = -1$ $t = -1$	M1 (attempt to solve) A1 (λ, μ) A1 (convincing)
Position vector of point of intersection is $\mathbf{i} - 2\mathbf{k}$	B1

(b)

$\cos\theta = \dfrac{(\mathbf{i}+\mathbf{j}+2\mathbf{k}).(\mathbf{i}+2\mathbf{j}+\mathbf{k})}{\lvert\mathbf{i}+\mathbf{j}+2\mathbf{k}\rvert\lvert\mathbf{i}+2\mathbf{j}+\mathbf{k}\rvert}$	B1 (identification of appropriate vectors) B1 (scalar product)
Now $(\mathbf{i}+\mathbf{j}+2\mathbf{k}).\ (\mathbf{i}+2\mathbf{j}+\mathbf{k})$ $= 1 + 2 + 2 = 5$	M1 (correct method) A1
$\lvert\mathbf{i}+\mathbf{j}+2\mathbf{k}\rvert = \sqrt{6}$ $\lvert\mathbf{i}+2\mathbf{j}+\mathbf{k}\rvert = \sqrt{6}$	B1 (for one)
$\cos\theta = \dfrac{5}{6}$	
$\theta \approx 34^{\circ}$.	B1

2005

[10]

9.	(a)	(i)	**OP** = **OA** + λ**AB**	M1
			$= 5\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k} + \lambda(-12\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k})$	B1 (**AB**)
			$\mathbf{r} = 5\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k} + \lambda(-12\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k})$	A1 (must have **r**, F.T. **AB**)
		(ii)	Point of intersection:	
			$5 - 12\lambda = -1 + 2\mu$	M1
			$1 + 3\lambda = 7 - 5\mu$	A1
			$\lambda = \frac{1}{3}, \mu = 1$	M1 A1 (CAO)
			Position vector = $\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$	B1 (candidate's parameters)

(b)	$(3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}).(\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k})$	M1
	$= 3 - 8 + 5 = 0$	
	∴ Vectors are perpendicular	A1 (value and conclusion)

[10]

2006

9.	*(a)*	**OP** = **OA** + λ **AB**	M1 (correct formula for r.h.s. and method for finding **AB**)
		= **i** + 3**j** + **k** + λ (**i** + 5**j** – 3**k**)	B1 (**AB**)
		r = **i** + 3**j** + **k** + λ (**i** + 5**j** – 3**k**)	A1 (must contain **r**, F.T. **AB**)
	(b)	(Point of intersection is on both lines) Equate coefficients of **i** and **j**	
		$1 + \lambda = 2 + \mu$	M1 (attempt to write equations using candidate's equations, one correct)
		$3 + 5\lambda = -1 + 2\mu$	A1 (two correct, using candidate's equations)
		$\lambda = -2,\ \mu = -3$	M1 (attempt to solve equations) A1 (C.A.O.)
		(Consider coefficients in **k**)	
		$p - \mu = 1 - 3\lambda$	M1 (use of equation in **k** to find p)
		$p = \mu + 1 - 3\lambda = 4$	A1 (F.T. candidate's λ, μ)
	(c)	**b** . **c** = (2**i** + 8**j** – 2**k**).(3**i** – **j** – **k**)	M1 (correct method)
		= 6 – 8 + 2 = 0	A1 (correct)
		b and **c** are ⊥^{r} vectors	A1 (C.A.O.)
			12

9.	*(a)*	(i)	$\mathbf{AB} = 3\mathbf{i} + 6\mathbf{j} + \mathbf{k} - (\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$		M1 ($\mathbf{b} - \mathbf{a}$)
			$= 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$		A1
		(ii)	Equation of AB is		M1 ($\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$)
			$\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})$	(o.e.)	A1 (must involve **r** or **OP** = ... F.T. **AB**)
		(iii)	(The point lies on both lines)		
			$\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})$		
			$= 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} + 4\mathbf{k})$		
			$1 + 2\lambda = 2 + \mu$		M1 (attempt to equate components, one correct)
			$3 + 3\lambda = 3 + \mu$		A1 (other correct)
					M1 (correct attempt to solve)
			$\lambda = -1,\ \mu = -3$		A1 (F.T. candidate's equations)
			Position vector of point of intersection is $-\mathbf{i} - 5\mathbf{k}$		A1 (C.A.O.)
	(b)		$\cos\theta = \dfrac{(\mathbf{i} + 2\mathbf{j} - \mathbf{k}).(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})}{\lvert\mathbf{i} + 2\mathbf{j} - \mathbf{k}\rvert\lvert 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}\rvert}$		M1 (attempt to use correct formula)
			$= \dfrac{1\times 3 - 2\times 1 - 1\times 2}{\sqrt{1^2 + 2^2 + (-1)^2}\sqrt{3^2 + 1^2 + 2^2}}$		M1 (correct attempt to find scalar product)
					A1 (scalar product)
			$= \dfrac{-1}{\sqrt{6}\sqrt{14}}$		B1 (one correct modulus)
					B1 (F.T. arithmetic slip in scalar product)
			$\theta = 96{\cdot}3°$ (accept nearest degree)		B1 (C.A.O.)
					15

2008

8.	*(a)(i)*	**AB** = **i** + 2**j** – 2**k**	B1 (**AB**)
	(ii)	Equation of AB is **r** = 4**i** – **j** +**k** + λ (**i** + 2**j** – 2**k**)	M1 (reasonable attempt to write equations) A1 (must contain **r**, F.T. candidate's **AB**)
	(b)	(Point of intersection is on both lines) Equate coeffs of **i** and **j** (any two of **i**,**j**,**k**)	M1 (attempt to write equations using candidate's equation)
		$1 + \mu = 4 + \lambda$	A1 (2 correct equations, F.T. candidate's equations)
		$\lambda = \frac{1}{3} \quad \left(\mu = \frac{10}{3}\right)$	M1 (attempt to solve) A1 (C.A.O.)
		Position vector is $\frac{13}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$	A1 (F.T. value of λ *or* μ)
	(c)	angle between $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{i} - \mathbf{j} + \mathbf{k}$ is required	B1 (coeffs of λ and μ)
		$\lvert \mathbf{i} + 2\mathbf{j} - 2\mathbf{k} \rvert = 3, \ \lvert \mathbf{i} - \mathbf{j} + \mathbf{k} \rvert = \sqrt{3}$	B1 (for one modulus)
		$(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}).(\mathbf{i} - \mathbf{j} + \mathbf{k}) = \lvert \quad \rvert \times \lvert \quad \rvert \cos\theta$	M1 (use of correct formula)
		$1 - 2 - 2 = 3\sqrt{3}\cos\theta$	B1 (l.h.s. unsimplified)
		$\theta = 125.3°$	A1 (C.A.O.)
			13

2009

8. *(a)* (i) $\mathbf{AB} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ B1

Equation of AB is

$\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} + 7\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ ($\mathbf{r} = \mathbf{a} + \lambda\mathbf{AB}$, o.e.) M1
A1

[Alternative:
$(1-\lambda)\mathbf{a} + \lambda\mathbf{b}$ (**a**, **b** substituted) M1
$\mathbf{r} = \ldots\ldots\ldots$ A1
(all correct) A1]

(ii) Assume AB and L intersect. Equate coefficients of **i**, **j** (o.e.).

$(3+\lambda) = 5 + 3\mu$ (F.T. candidate's values) M1
$4 - 2\lambda = 6 - 2\mu$ A1

Solve for λ, μ, (attempt to solve for λ, μ) m1

$\lambda = -\frac{5}{2}$, $\mu = -\frac{3}{2}$ (one value; F.T. one slip) A1

Check **k** coefficient (o.e.)

L.H.S. $= 7 + 3\lambda = -\frac{1}{2}$ (attempt to check) m1

R.H.S. $= 1 + \mu = -\frac{1}{2}$

(Terms check so lines intersect)

Point of intersection is $\mathbf{i} + 9\mathbf{j} - \frac{1}{2}\mathbf{k}$. C.A.O. A1

(dependent on M1, m1 earlier)

(b) $(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}).(2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 0$ (correct method of finding scalar product) M1
$6 - 2 - 4 = 0$ A1
(therefore vectors are perpendicular)

2010

20

9. (*a*) $(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}).(\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}) = 18$ B1

$|2\mathbf{i} - 2\mathbf{j} + \mathbf{k}| = \sqrt{9}$, $|\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}| = \sqrt{81}$ (one correct) B1

Correctly substituting in the formula $\mathbf{a.b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos\theta$ M1

$\theta = 48{\cdot}2°$ (c.a.o.) A1

(*b*) (i) $\mathbf{AB} = -\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$ B1

(ii) Use of $\mathbf{a} + \lambda\mathbf{AB}$, $\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$, $\mathbf{b} + \lambda\mathbf{AB}$ or $\mathbf{b} + \lambda(\mathbf{b} - \mathbf{a})$ to find vector equation of *AB* M1

$\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} - 2\mathbf{j} + 7\mathbf{k})$ (o.e.)

(f.t. if candidate uses his/her expression for **AB**) A1

(*c*) $2 - \lambda = -1 + \mu$

$-2 - 2\lambda = -4 + \mu$

$1 + 7\lambda = -2 - \mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving two of the equations simultaneously m1

(f.t. for all 3 marks if candidate uses his/her expression for **AB**)

$\lambda = -1$, $\mu = 4$ (o.e.) (c.a.o.) A1

Correct verification that values of λ and μ satisfy third equation A1

Position vector of point of intersection is $3\mathbf{i} - 6\mathbf{k}$ (f.t. one slip) A1

2011

9. (*a*) Use of $(5\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}).(4\mathbf{i} + 6\mathbf{j} + a\mathbf{k}) = 0$ M1

$5 \times 4 + (-8) \times 6 + 4 \times a = 0$ m1

$a = 7$ A1

(*b*) (i) $\mathbf{r} = 8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ (o.e.) B1

(ii) $8 + 2\lambda = 4 - 2\mu$

$3 + \lambda = 7 + \mu$

$-7 + 2\lambda = 5 + 3\mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving two of the equations simultaneously m1

$\lambda = 1$, $\mu = -3$ (o.e.) (c.a.o.) A1

Correct verification that values of λ and μ do not satisfy third equation B1

2012

9. (*a*) An attempt to evaluate **a.b** M1
Correct evaluation of **a.b** and **a.b** $\neq 0 \Rightarrow$ **a** and **b** not perpendicular A1

(*b*) (i) **AB** = 2**i** + **j** + 2**k** B1
(ii) Use of **a** + λ**AB**, **a** + λ(**b** – **a**), **b** + λ**AB** or **b** + λ(**b** – **a**) to find vector equation of AB M1
r = 4**i** + **j** – 6**k** + λ (2**i** + **j** + 2**k**) (o.e.)
(f.t. if candidate uses his/her expression for **AB**) A1

(*c*) $4 + 2\lambda = 2 - 2\mu$
$1 + \lambda = 6 + \mu$
$-6 + 2\lambda = p + 3\mu$ (o.e.)
(comparing coefficients, at least one equation correct) M1
(at least two equations correct) A1
Solving the first two of the equations simultaneously m1
(f.t. for all 3 marks if candidate uses his/her expression for **AB**)
$\lambda = 2$, $\mu = -3$ (o.e.) (c.a.o.) A1
$p = 7$ from third equation
(f.t. candidates derived values for λ and μ) A1

2013

9. (*a*) **AB** = 8**i** – 4**j** + 12**k** B1

(*b*) **OC** = – **i** + 3**j** – 7**k** + $^3/_4$(8**i** – 4**j** + 12**k**) (o.e.) M1
OC = 5**i** + 2**k** A1

(*c*) (i) Use of **OA** + μ(– 4**i** + **j** + 3**k**) on r.h.s. M1
r = – **i** + 3**j** – 7**k** + μ(– 4**i** + **j** + 3**k**) (all correct) A1
(ii) $-1 + \lambda \times (-4) = 7$
(an equation in λ from one set of coefficients) M1
$\lambda = -2$ A1
$1 + (-2) \times 1 = -1$
$11 + (-2) \times 3 = 5$ (both verifications) A1
An attempt to evaluate **AB**.(– 4**i** + **j** + 3**k**) M1
AB.(– 4**i** + **j** + 3**k**) = 0 (convincing) A1
B lies on L, AB is perpendicular to L and thus B is the foot of the perpendicular from A to L (c.a.o.) A1

9. (*a*) $\mathbf{p.q} = -18$ B1

$|\mathbf{p}| = \sqrt{14}$, $|\mathbf{q}| = \sqrt{105}$ (at least one correct) B1

Correctly substituting candidate's derived values in the formula

$\mathbf{p.q} = |\mathbf{p}| \times |\mathbf{q}| \times \cos\theta$ M1

$\theta = 118°$ (c.a.o.) A1

(*b*) (i) Use of $\mathbf{CD} = \mathbf{CO} + \mathbf{OD}$ and the fact that $\mathbf{OC} = \frac{1}{2}\mathbf{b}$ and $\mathbf{OD} = 2\mathbf{a}$, leading to printed answer $\mathbf{CD} = 2\mathbf{a} - \frac{1}{2}\mathbf{b}$

(convincing) B1

Use of $\frac{1}{2}\mathbf{b} + \lambda\mathbf{CD}$ (o.e.) to find vector equation of *CD* M1

Vector equation of *CD*: $\mathbf{r} = 2\lambda\mathbf{a} + \frac{1}{2}(1 - \lambda)\mathbf{b}$ (convincing) A1

(ii) **Either:**

Either substituting $\frac{1}{3}$ for λ in the vector equation of *CD*

or substituting 2 for μ in the vector equation of *L* M1

At least one of these position vectors $= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ A1

Both position vectors $= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} \Rightarrow$ this must be the position vector of the point of intersection *E* A1

Or:

$2\lambda = \frac{\mu}{3}$

$\frac{1}{2}(1 - \lambda) = \frac{1}{3}(\mu - 1)$

(comparing candidate's coefficients of **a** and **b** and an attempt to solve) M1

$\lambda = \frac{1}{3}$ or $\mu = 2$ A1

$\mathbf{OE} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ (convincing) A1

(iii) **Either**: *E* lies on *AB* and is such that *AE* : *EB* = 1 : 2 (o.e.)

Or: *E* is the point of intersection of *AB* and *CD* B1

2015

8. (*a*) (i) $\mathbf{AB} = -\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$ B1

(ii) Use of $\mathbf{a} + \lambda\mathbf{AB}$, $\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$, $\mathbf{b} + \lambda\mathbf{AB}$ or $\mathbf{b} + \lambda(\mathbf{b} - \mathbf{a})$ to find vector equation of AB M1

$\mathbf{r} = 5\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} - 2\mathbf{j} + 7\mathbf{k})$ (o.e.)

(f.t. if candidate uses his/her expression for **AB**) A1

(*b*) $5 - \lambda = 2 + \mu$

$-1 - 2\lambda = -3 + \mu$

$-1 + 7\lambda = -4 - \mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving two of the equations simultaneously m1

(f.t. for all 3 marks if candidate uses his/her equation of AB)

$\lambda = -1$, $\mu = 4$ (o.e.) (c.a.o.) A1

Correct verification that values of λ and μ satisfy third equation A1

Position vector of point of intersection is $6\mathbf{i} + \mathbf{j} - 8\mathbf{k}$

2016

8. (*a*) (i) $\mathbf{AB} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ B1

(ii) Use of $\mathbf{a} + \lambda\mathbf{AB}$, $\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$, $\mathbf{b} + \lambda\mathbf{AB}$ or $\mathbf{b} + \lambda(\mathbf{b} - \mathbf{a})$ to find vector equation of AB M1

$\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 3\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ (o.e.)

(f.t. if candidate uses his/her expression for **AB**) A1

(*b*) (i) $1 + 2\lambda = -1 - 2\mu$

$3 + \lambda = 8 + \mu$

$-3 + 2\lambda = p + 3\mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving the first two equations simultaneously m1

(f.t. for all 3 marks if candidate uses his/her expression for **AB**)

$\lambda = 2$, $\mu = -3$ (o.e.) (c.a.o.) A1

$p = 10$ from third equation (f.t. candidate's derived values for λ and μ provided the third equation is correct) A1

(ii) An attempt to evaluate $(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}).(6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ M1

$(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}).(6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}) = -1 \neq 0 \Rightarrow L$ and $(6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ not perpendicular A1

9.	(*a*)	$\mathbf{AD} = \mathbf{AO} + \mathbf{OD} = -\mathbf{a} + 2\mathbf{b}$	B1
		Use of $\mathbf{a} + \lambda\mathbf{AD}$ (o.e.) to find vector equation of AD	M1
		Vector equation of AD: $\mathbf{r} = \mathbf{a} + \lambda(-\mathbf{a} + 2\mathbf{b})$	
		$\mathbf{r} = (1 - \lambda)\mathbf{a} + 2\lambda\mathbf{b}$ (convincing)	A1
	(*b*)	$\mathbf{BC} = \mathbf{BO} + \mathbf{OC} = 5\mathbf{a} - \mathbf{b}$	B1
		Vector equation of BC: $\mathbf{r} = \mathbf{b} + \mu(5\mathbf{a} - \mathbf{b})$	
		$\mathbf{r} = 5\mu\mathbf{a} + (1 - \mu)\mathbf{b}$ (o.e.)	B1
	(*c*)	$1 - \lambda = 5\mu$	
		$2\lambda = 1 - \mu$	
		(comparing candidate's coefficients of **a** and **b** and an attempt to solve)	M1
		$\lambda = \frac{4}{9}$ or $\mu = \frac{1}{9}$ (f.t. candidate's derived vector equation of BC)	A1
		$\mathbf{OE} = \frac{5}{9}\mathbf{a} + \frac{8}{9}\mathbf{b}$ (f.t. candidate's derived vector equation of BC)	A1