

C4 Vector Questions

Specimen

9. The vector equations of two lines are

$$\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \lambda(\mathbf{i} + \mathbf{j} + 2\mathbf{k}),$$

$$\mathbf{r} = 2\mathbf{i} + 2\mathbf{j} + t\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k}),$$

where t is a constant.

- (a) Given that the two lines intersect, show that $t = -1$ and find the position vector of the point of intersection. [6]
- (b) Find the acute angle between the lines, giving your answer correct to the nearest degree. [6]

2005

9. (a) The position vectors of the points A and B are given by $\mathbf{a} = 5\mathbf{i} + \mathbf{j} + 2\mathbf{k}$,
 $\mathbf{b} = -7\mathbf{i} + 4\mathbf{j} - \mathbf{k}$.

- (i) Find the vector equation of the line AB . [3]
- (ii) The vector equation of the line L is

$$\mathbf{r} = -\mathbf{i} + 7\mathbf{j} + 8\mathbf{k} + \mu(2\mathbf{i} - 5\mathbf{j} - 7\mathbf{k}).$$

Given that AB and L intersect, find the position vector of the point of intersection. [5]

- (b) Show that the vectors $\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ are perpendicular. [2]

2006

9. The position vectors of the points A and B are given by

$$\mathbf{a} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}.$$

- (a) Find the vector equation of the line AB . [3]
- (b) The vector equation of the line L is

$$\mathbf{r} = 2\mathbf{i} - \mathbf{j} + p\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$$

where p is a constant. Given that AB and L intersect, find the value of p . [6]

- (c) Given $\mathbf{c} = 3\mathbf{i} - \mathbf{j} - \mathbf{k}$, find $\mathbf{b} \cdot \mathbf{c}$. What does your answer tell you about the vectors \mathbf{b} and \mathbf{c} ? [3]

2007

9. (a) The position vectors of the points A and B , relative to a fixed origin O , are $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $3\mathbf{i} + 6\mathbf{j} + \mathbf{k}$, respectively.
- (i) Find \mathbf{AB} .
 - (ii) Find the vector equation of the line AB .
 - (iii) The vector equation of the line L is $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.
Given that L and AB intersect, find the position vector of the point of intersection. [9]
- (b) Find the angle between the vectors $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. [6]

2008

8. The position vectors of the points A and B are given by

$$\mathbf{a} = 4\mathbf{i} - \mathbf{j} + \mathbf{k}, \quad \mathbf{b} = 5\mathbf{i} + \mathbf{j} - \mathbf{k}.$$

- (a) (i) Write down the vector \mathbf{AB} .
(ii) Find the vector equation of the line AB . [3]

The vector equation of the line L is

$$\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 3\mathbf{k} + \mu(\mathbf{i} - \mathbf{j} + \mathbf{k}).$$

- (b) Given that the lines AB and L intersect, find the position vector of the point of intersection. [5]
(c) Find the angle between the line AB and the line L . [5]

2009

8. (a) The position vectors of the points A and B are given by

$$\mathbf{a} = 3\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}, \quad \mathbf{b} = 4\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}.$$

- (i) Find the vector equation of the line AB .
- (ii) The vector equation of the line L is

$$\mathbf{r} = 5\mathbf{i} + 6\mathbf{j} + \mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} + \mathbf{k}).$$

Show that AB and L intersect and find the position vector of the point of intersection. [9]

- (b) Show that the vectors $3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ are perpendicular. [2]

2010

9. The position vectors of the points A and B are given by

$$\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k},$$
$$\mathbf{b} = \mathbf{i} - 4\mathbf{j} + 8\mathbf{k},$$

respectively.

- (a) Find the angle between the vectors \mathbf{a} and \mathbf{b} . [4]

- (b) (i) Write down the vector \mathbf{AB} .

- (ii) Find the vector equation of the line AB . [3]

- (c) The vector equation of the line L is given by

$$\mathbf{r} = -\mathbf{i} - 4\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k}).$$

Show that the lines AB and L intersect and find the position vector of the point of intersection. [6]

2011

9. (a) Given that the vectors $5\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}$ and $4\mathbf{i} + 6\mathbf{j} + a\mathbf{k}$ are perpendicular, find the value of the constant a . [3]

- (b) The line L_1 passes through the point with position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$ and is parallel to the vector $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

- (i) Write down the vector equation of the line L_1 .

- (ii) The line L_2 has vector equation

$$\mathbf{r} = 4\mathbf{i} + 7\mathbf{j} + 5\mathbf{k} + \mu(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}).$$

Show that L_1 and L_2 do not intersect. [6]

2012

9. The position vectors of the points A and B are given by

$$\mathbf{a} = 4\mathbf{i} + \mathbf{j} - 6\mathbf{k},$$
$$\mathbf{b} = 6\mathbf{i} + 2\mathbf{j} - 4\mathbf{k},$$

respectively.

- (a) Determine whether or not the vectors \mathbf{a} and \mathbf{b} are perpendicular, giving a reason for your answer. [2]

- (b) (i) Write down the vector \mathbf{AB} .

- (ii) Find the vector equation of the line AB . [3]

- (c) The vector equation of the line L is given by

$$\mathbf{r} = 2\mathbf{i} + 6\mathbf{j} + p\mathbf{k} + \mu(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}),$$

where p is a constant.

Given that the lines AB and L intersect, find the value of p . [5]

2013

9. The position vectors of the points A and B are given by

$$\begin{aligned}\mathbf{a} &= -\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}, \\ \mathbf{b} &= 7\mathbf{i} - \mathbf{j} + 5\mathbf{k},\end{aligned}$$

respectively.

(a) Write down the vector \mathbf{AB} . [1]

(b) The point C lies on the line AB and is such that $AC : CB = 3 : 1$.
Find the position vector of C . [2]

(c) The vector equation of the line L is given by

$$\mathbf{r} = -\mathbf{i} + \mathbf{j} + 11\mathbf{k} + \lambda(-4\mathbf{i} + \mathbf{j} + 3\mathbf{k}).$$

- (i) Find the vector equation of the line parallel to L which passes through A .
(ii) Verify that B is in fact the foot of the perpendicular from A to L . [8]



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2014

9. (a) The vectors \mathbf{p} and \mathbf{q} are given by

$$\mathbf{p} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$$

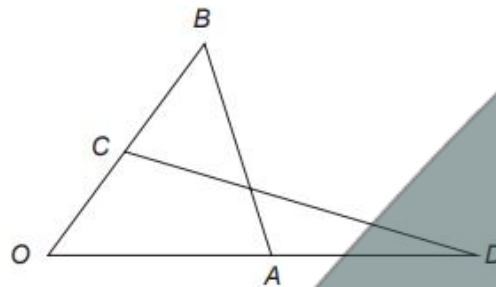
and

$$\mathbf{q} = 5\mathbf{i} + 4\mathbf{j} - 8\mathbf{k}.$$

Find the angle between \mathbf{p} and \mathbf{q} .

[4]

- (b) In the diagram below, the points O , A , B , C and D are such that A is the mid-point of OD and C is the mid-point of OB .



Taking O as the origin, the position vectors of A and B are denoted by \mathbf{a} and \mathbf{b} respectively.

- (i) Show that $\mathbf{CD} = 2\mathbf{a} - \frac{1}{2}\mathbf{b}$.

Hence show that the vector equation of the line CD may be expressed in the form

$$\mathbf{r} = 2\lambda\mathbf{a} + \frac{1}{2}(1 - \lambda)\mathbf{b}.$$

The vector equation of the line L may be expressed in the form

$$\mathbf{r} = \frac{1}{3}\mu\mathbf{a} + \frac{1}{3}(\mu - 1)\mathbf{b}.$$

The lines CD and L intersect at the point E .

- (ii) By giving λ and μ appropriate values, or otherwise, show that E has position vector $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$.

- (iii) Give a geometrical interpretation of the fact that E has position vector $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$. [7]

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2015

8. The position vectors of the points A and B are given by

$$\begin{aligned}\mathbf{a} &= 5\mathbf{i} - \mathbf{j} - \mathbf{k}, \\ \mathbf{b} &= 4\mathbf{i} - 3\mathbf{j} + 6\mathbf{k},\end{aligned}$$

respectively.

- (a) (i) Write down the vector \mathbf{AB} .
(ii) Find the vector equation of the line AB . [3]
- (b) The vector equation of the line L is given by

$$\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k}).$$

Show that the lines AB and L intersect and find the position vector of the point of intersection. [6]

2016

8. The position vectors of the points A and B are given by

$$\begin{aligned}\mathbf{a} &= \mathbf{i} + 3\mathbf{j} - 3\mathbf{k}, \\ \mathbf{b} &= 3\mathbf{i} + 4\mathbf{j} - \mathbf{k},\end{aligned}$$

respectively.

- (a) (i) Write down the vector \mathbf{AB} .
(ii) Find the vector equation of the line AB . [3]
- (b) The vector equation of the line L is given by

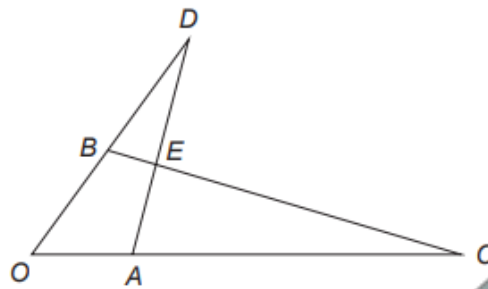
$$\mathbf{r} = -\mathbf{i} + 8\mathbf{j} + p\mathbf{k} + \mu(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}),$$

where p is a constant.

- (i) Given that the lines AB and L intersect, find the value of p .
(ii) Determine whether or not the line L is perpendicular to the vector $6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$, giving a reason for your answer. [7]

2017

9. In the diagram below, the points O, A, B, C and D are as follows. A lies on OC and $OC = 5OA$. B lies on OD and $OD = 2OB$.



Taking O as origin, the position vectors of A and B are denoted by \mathbf{a} and \mathbf{b} respectively.

- (a) Write down the vector \mathbf{AD} in terms of \mathbf{a} and \mathbf{b} .
Hence show that the vector equation of the line AD may be expressed in the form

$$\mathbf{r} = (1 - \lambda)\mathbf{a} + 2\lambda\mathbf{b}. \quad [3]$$

- (b) Find a similar expression for the vector equation of the line BC . [2]

- (c) The lines AD and BC intersect at the point E . Find the position vector of E in terms of \mathbf{a} and \mathbf{b} . [3]