C4 Volumes of Rotation Answers

Specimen

7. Volume $= \pi\int_1^e x^3 \ln x\,dx$ B1

$= \pi\left\{\left[\frac{x^4}{4}\ln x\right]_1^e - \int_1^e \frac{x^4}{4}\frac{1}{x}\,dx\right\}$ M1 (parts, correct choice of u, v)

A1

$= \pi\left\{\left[\frac{x^4}{4}\ln x\right]_1^e - \int_1^e \frac{x^3}{4}\,dx\right\}$ m1 (division)

$= \pi\left\{\left[\frac{x^4}{4}\ln x\right]_1^e - \left[\frac{x^4}{16}\right]_1^e\right\}$ A1

$= \frac{\pi}{16}(3e^4 + 1)$ A1

2005

5. Volume $= \pi\int_1^4 \left(\sqrt{x} + \frac{4}{\sqrt{x}}\right)^2 dx$ B1

$= \pi\int_1^4 \left(x + 8 + \frac{16}{x}\right)dx$ B1

$= \pi\left[\frac{x^2}{2} + 8x + 16\ln x\right]_1^4$ B1

$= \pi\left(31\frac{1}{2} + 16\ln 4\right)$ M1

$\approx$ 168.6 A1 (C.A.O.) [5]

2006

5. Volume $= \pi\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$ B1

$= (\pi)\int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}dx$ M1 ($a + b\cos 2x$)

A1 ($a = \frac{1}{2}, b = \frac{1}{2}$)

$= (\pi)\left[\frac{x}{2} - \frac{\sin 2x}{4}\right]$ A1 $\left(\frac{x}{2} - \frac{\sin 2x}{4}\right)$ (F.T. a, b))

$= \frac{\pi^2}{4}$ ($\approx$ 2·467, accept 2·47, 3 sig. figures) A1 (C.A.O.)

2007

5. Volume $= \pi \int_0^1 (e^{2x} + 1)\ dx$ — B1 (with or without limits, after squaring $\sqrt{\ }$)

$= \pi \left[\frac{e^{2x}}{2} + x \right]_0^1$ — B1 (correct integration)

$= \pi \left[\frac{e^{2x}}{2} + 1 - \frac{1}{2} \right]$ — M1 (correct use of limits after attempted integration)

$\approx 13{\cdot}177$ — A1 (C.A.O.)

2008

4. Volume $= \pi \int_1^4 \left(x + \frac{3}{\sqrt{x}} \right)^2 dx$ — B1

$= \pi \int_1^4 \left(x^2 + 6\sqrt{x} + \frac{9}{x} \right) dx$ — M1 (attempt to square, at least 2 correct terms) A1 (all correct)

$= \pi \left[\frac{x^3}{3} + 4x^{\frac{3}{2}} + 9\ln x \right]_1^4$ — A3 (integration of 3 terms, F.T. similar work $Ax^2 + B\sqrt{x} + \frac{c}{x}$)

$= \pi[49 + 9\ln x] \approx 193.1$ or 61.48π — A1 (C.A.O.)

7

2009

4. Volume $= \pi\int_0^{\frac{\pi}{8}} \cos^2 2x \, dx$ (must contain limits) B1

$= (\pi)\int_0^{\frac{\pi}{8}} \frac{1+\cos 4x}{2} \, dx$ $(\cos^2 2x = a + b\cos 4x; \quad a, b \neq 0)$ M1

$\left(a = \frac{1}{2}, b = \frac{1}{2}\right)$ A1

$= (\pi)\left[\frac{x}{2} + \frac{\sin 4x}{8}\right]_0^{\frac{\pi}{8}}$ A1

$= (\pi)\left(\frac{\pi}{16} + \frac{1}{8} - 0 - 0\right)$ (correct use of limits) m1

$= \frac{\pi}{2}\left(\frac{\pi}{8} + \frac{1}{4}\right)$ or 1·0095 (C.A.O.) A1

[If substitution used, marks are gained after

$\frac{1}{2}\cos^2 u = a + b\cos 2u$ M1]

2010

4.

Volume $= \pi\int_{\pi/6}^{\pi/3} \sin^2 x \, dx$ B1

Use of $\sin^2 x = \frac{(\pm 1 \pm \cos 2x)}{2}$ M1

Correct integration of candidate's $\frac{(\pm 1 \pm \cos 2x)}{2}$ A1

Correct substitution of correct limits in candidate's integrated expression M1

Volume $= \frac{\pi^2}{12} = 0{\cdot}822(467...)$ (c.a.o.) A1

2011

5. (*a*) $A(-3, 0), B(3, 0), C(0, 3)$ B1

(*b*) (i) $\text{Volume} = \pi \int_{-3}^{3} (9 - x^2)\,dx$

(f.t candidate's x-coordinates for A, B) M1

$\int (9 - x^2)\,dx = 9x - \frac{x^3}{3}$ B1

$\text{Volume} = 36\pi$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

(ii) This is the volume of a sphere of radius 3 E1

2012

4.

$\text{Volume} = \pi \int_{3}^{4} \left(\sqrt{x} + \frac{5}{\sqrt{x}}\right)^2 dx$ B1

$\left(\sqrt{x} + \frac{5}{\sqrt{x}}\right)^2 = \left(x + 10 + \frac{25}{x}\right)$ B1

$\int \left(ax + b + \frac{c}{x}\right) dx = \frac{ax^2}{2} + bx + c \ln x$, where $c \neq 0$ and at least one of $a, b \neq 0$ B1

Correct substitution of correct limits in candidate's integrated expression of form $\frac{ax^2}{2} + bx + c \ln x$, where $c \neq 0$ and at least one of $a, b \neq 0$ M1

$\text{Volume} = 65(\cdot 0059\ldots)$ (c.a.o.) A1

2013

4.

Volume $= \pi \int_{\pi/6}^{\pi/2} \sin^2 2x \, dx$ B1

$\sin^2 2x = \frac{(1 - \cos 4x)}{2}$ B1

$\int (a + b\cos 4x)\, dx = ax + \frac{1}{4} b \sin 4x, \quad a \neq 0, b \neq 0$ B1

Correct substitution of candidate's limits in candidate's integrated expression of form $mx + n \sin 4x \quad m \neq 0, n \neq 0$ M1

Volume = 1·985 (c.a.o.) A1

Note: Answer only with no working earns 0 marks

2014

4.

Volume $= \pi \int_{0}^{\pi/4} (3 + 2\sin x)^2 \, dx$ B1

Correct use of $\sin^2 x = \frac{(1 - \cos 2x)}{2}$ M1

Integrand $= (9 + 2 + 12\sin x - 2\cos 2x)$ (c.a.o.) A1

$\int (a + b\sin x + c\cos 2x)\, dx = (ax - b\cos x + \frac{c}{2}\sin 2x) \quad (a \neq 0, b \neq 0, c \neq 0)$ B1

Correct substitution of correct limits in candidate's integrated expression of form $(ax - b\cos x + \frac{c}{2}\sin 2x) \quad (a \neq 0, c \neq 0)$ M1

Volume = 35 (c.a.o.) A1

Note: Answer only with no working earns 0 marks

2015

4. (*a*)

$V = \pi \int_0^a (mx)^2 \, dx$ M1

$\int (mx)^2 \, dx = \frac{m^2x^3}{3}$ B1

$V = \frac{\pi m^2 a^3}{3}$ (c.a.o.) A1

(*b*) (i) Substituting $\frac{b}{a}$ for m in candidate's derived expression for V M1

$V = \frac{\pi b^2 a}{3}$ (c.a.o.) A1

(ii) This is the volume of a cone of (vertical) height a and (base) radius b E1

2016

9.

Volume $= \pi \int_{\pi/5}^{2\pi/5} (\cos x + \sin x)^2 \, dx$ B1

$(\cos x + \sin x)^2 = \cos^2 x + \sin^2 x + 2 \sin x \cos x$ B1

$\int (\cos^2 x + \sin^2 x) \, dx = x$ **or** $\left[\frac{x}{2} + \frac{1}{4} \sin 2x\right] + \left[\frac{x}{2} - \frac{1}{4} \sin 2x\right]$ B1

$\int k \sin x \cos x \, dx = -\frac{k}{4} \cos 2x$ **or** $\frac{k}{2} \sin^2 x$ **or** $-\frac{k}{2} \cos^2 x$ B1

Substitution of limits in candidate's integrated expression
(awarded only if at least two of the previous three marks have been awarded) M1

Volume = 3·73 (c.a.o.) A1

Note: Answer only with no working earns 0 marks

4.

Volume $= \pi \int_{\pi/6}^{\pi/3} (\cos x + \sec x)^2 \, dx$ B1

Correct use of $\cos^2 x = \frac{(1 + \cos 2x)}{2}$ M1

Integrand $= \frac{(1 + \cos 2x)}{2} + 2 + \sec^2 x$ (c.a.o.) A1

$\int a \cos 2x \, dx = \frac{a}{2} \sin 2x$ $(a \neq 0)$ B1

$\int b \, dx = bx$ and $\int \sec^2 x \, dx = \tan x$ $(b \neq 0)$ B1

Correct substitution of correct limits in candidate's integrated expression of the form

$px + q\sin 2x + \tan x$ $(p \neq 0, q \neq 0)$ M1

Volume $= \pi \times (4{\cdot}566551037 - 2{\cdot}102853559) = 7{\cdot}74$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks